Nonlinear Analysis 74 (2011) 1261–1271

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

# Limit cycles for generalized Kukles polynomial differential systems

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ARTICLE INFO

*Article history:*
Received 24 November 2009
Accepted 29 September 2010

*MSC:*
34C29
34C25
47H11

*Keywords:*
Limit cycle
Kukles systems
Averaging theory

ABSTRACT

We study the limit cycles of generalized Kukles polynomial differential systems using averaging theory of first and second order.



## 1. Introduction

One of the main problems in the qualitative theory of real planar differential equations is the determination of limit cycles. Limit cycles of planar vector fields were defined by Poincaré [1]. At the end of the 1920s van der Pol [2], Liénard [3] and Andronov [4] proved that a closed orbit of a self-sustained oscillation occurring in a vacuum tube circuit was a limit cycle as considered by Poincaré. After these works, the non-existence, existence, uniqueness and other properties of limit cycles were studied extensively by mathematicians and physicists, and more recently also by chemists, biologists, economists, etc. (see for instance the books [5,6]).

The second part of the sixteenth Hilbert problem [7] is related to the least upper bound on the number of limit cycles of polynomial vector fields having a fixed degree. This problem and the Riemann conjecture are the only two problems on the list of Hilbert which have not been solved. Here we consider a very particular case of the sixteenth Hilbert problem; we want to study the upper bound of the generalized Kukles polynomial differential system

$$\dot{x} = -y, \qquad \dot{y} = Q(x, y), \tag{1}$$

where $Q(x, y)$ is a polynomial with real coefficients of degree $n$. This system was introduced by Kukles in [8], giving necessary and sufficient conditions in order that the system

$$\dot{x} = -y, \qquad \dot{y} = x + a_0 y + a_1 x^2 + a_2 xy + a_3 y^2 + a_4 x^3 + a_5 x^2 y + a_6 xy^2 + a_7 y^3, \tag{2}$$

has a center at the origin.

Recently the question of the number of limit cycles of these systems has attracted increasing interest. In [9] Sadovskii solves the center–focus problem for system (2) with $a_2 a_7 \neq 0$ and proves that systems (2) can have seven limit cycles.

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0362-546X/$ – see front matter 
doi:10.1016/j.na.2010.09.064