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A second-order estimate for blow-up solutions of elliptic equations

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ABSTRACT

We investigate second-term asymptotic behavior of boundary blow-up solutions to the problems $\Delta u = b(x)f(u), x \in \Omega$, subject to the singular boundary condition $u(x) = \infty$, in a bounded smooth domain $\Omega \subset R^N$. $b(x)$ is a non-negative weight function. The nonlinearly f is regularly varying at infinity with index $\rho > 1$ (that is $\lim_{u\to\infty} f(\xi u)/f(u) = \xi^\rho$ for every $\xi > 0$) and the mapping $f(u)/u$ is increasing on $(0, +\infty)$. The main results show how the mean curvature of the boundary $\partial\Omega$ appears in the asymptotic expansion of the solution $u(x)$. Our analysis relies on suitable upper and lower solutions and the Karamata regular variation theory.

1. Introduction and main results

Let $\Omega \subset R^N$ $(N \geq 3)$ be a bounded domain with a smooth boundary; we are interested in the second-order asymptotic behavior of the boundary blow-up solutions to the elliptic problems

$$\begin{cases} \Delta u = b(x)f(u), & x \in \Omega, \\ u(x) = \infty, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where $b \in C^{0,\mu}(\overline{\Omega})$ $(\mu \in (0, 1))$ is non-negative and satisfies

(b_1) $b(x) = k^2(d(x))(1 + cd^\theta + o(d^\theta))$ for some $k \in \mathcal{K}_l$ with $l > 0$,

where $c, \theta > 0$ are constants, and $d(x) = \mathrm{dist}(x, \partial\Omega)$ for each $x \in \Omega$. $\mathcal{K}_l$ denotes the set of all positive non-decreasing functions $k \in L^1(0, \vartheta) \cap C^1(0, \vartheta)$ which satisfy

$$\lim_{t\to 0+} \frac{K(t)}{k(t)} = 0, \qquad \lim_{t\to 0+} \frac{\mathrm{d}}{\mathrm{d}t}\left(\frac{K(t)}{k(t)}\right) = l, \quad \text{where } K(t) = \int_0^t k(s)\mathrm{d}s.$$

For more propositions for $\mathcal{K}_l$, refer to [1–3].

Throughout this paper, we will make the following assumption on the nonlinearly f:

(f_1) $f \in C^1[0, +\infty)$ and $f(u)/u$ is increasing on $(0, \infty)$;
(f_2) f satisfies the Keller–Osserman condition [4,5],

$$\int_1^\infty \frac{\mathrm{d}t}{\sqrt{F(t)}} < \infty, \quad \text{where } F(t) = \int_0^t f(s)\mathrm{d}s.$$

The boundary condition $u(x) = \infty, x \in \partial\Omega$ is to be understand as $u \to \infty$ when $d(x) = \mathrm{dist}(x, \partial\Omega) \to 0+$. The solutions of problem (1.1) are called large solutions, boundary blow-up solutions or explosive solutions.

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