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Fixed point theorems for generalized contractions in partially ordered metric spaces with semi-monotone metric[☆]

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ABSTRACT

In this paper, we study the existence and uniqueness of (coupled) fixed points for mixed monotone mappings in partially ordered metric spaces with semi-monotone metric. As an application, we prove the existence and uniqueness of the solution for a first-order differential equation with periodic boundary conditions.

1. Introduction

In recent years, there has been an increasing interest in the study of fixed points for mappings that possess monotonicity type properties, in the context of partially ordered metric spaces (cf. [1–18]). The used approach was to combine some contraction principle (e.g., Banach's contraction principle, or any of its many generalizations) with the method of monotone iterations and the method of upper and lower solutions, while weakening the contraction condition. Following this trend, Gnana Bhaskar and Lakshmikantham [3], Drici et al. [6], Lakshmikantham and Ćirić [10], Samet [16], Choudhury and Kundu [13] investigated the existence and the uniqueness of fixed points and coupled fixed points of mixed monotone mappings in partially ordered metric spaces and obtained important results which were then applied to the study of several nonlinear problems.

Recall that if $(X, \preceq)$ is a partially ordered set, a bivariate mapping $A : X \times X \to X$ is said to be mixed monotone (or is said to have the mixed monotone property) (cf. [19,3]) if A is nondecreasing in the first argument and nonincreasing in the second argument, i.e.,

$$x_1, x_2, y \in X, \quad x_1 \preceq x_2 \Rightarrow A(x_1, y) \preceq A(x_2, y)$$

and

$$x, y_1, y_2 \in X, \quad y_1 \preceq y_2 \Rightarrow A(x, y_1) \succeq A(x, y_2).$$

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