



Comparison of optimal portfolios with and without subsistence consumption constraints

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ABSTRACT

We present the effects of the subsistence consumption constraints on a portfolio selection problem for an agent who is free to choose when to retire with a constant relative risk aversion (CRRA) utility function. By comparing the previous studies with and without the constraints expressed by the minimum consumption requirement, the changes of a retirement wealth level and the amount of money invested in the risky asset are derived explicitly. As a result, the subsistence constraints always lead to lower retirement wealth level but do not always induce less investment in the risky asset. This implies that even though the agent who has a restriction on consumption retires with lower wealth level, she invests more money near the retirement when her risk aversion lies inside a certain range.

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1. Introduction

There has been the endeavor to investigate a realistic model of portfolio optimization after the seminal works of Merton [1,2]. Early studies of portfolio theory are mainly about realistic constraints. Especially, intertemporal optimal consumption and investment problems with these constraints have been widely considered for academic and practical reasons. In addition, the voluntary retirement problem is indispensable to a working agent in these days. The economic agent who receives a wage income faces disutility about his labor and hence wants to retire as soon as possible. The model with realistic constraints and voluntary retirement has a benefit to explain the realistic behavior of an economic agent but, in many cases, it usually is an intractable problem.

There are mainly two types of constraints in portfolio theory. One is a consumption constraint and the other is a portfolio constraint. Cheng and Wei [3] investigated the Merton's problem with habit formation and obtained the optimal consumption. Moreover, they also found the relation between the optimal investment with habit and constant proportion portfolio insurance (CPPI) for a special habit. The subsistence constraint is considered as one of the habit formations, which represents the minimum consumption level for living. This constraint is defined as the inequality that an agent's consumption rate should be greater than or equal to a certain positive constant. In [4–8], they solved the portfolio optimization problem with subsistence constraints. There are also many types of portfolio constraints but we do not mention these constraints because we only have our attention on consumption constraints, especially, the subsistence constraints.

The voluntary retirement problem is formulated as an optimal stopping time problem which is broadly applied to finance. To resolve the stopping time problem, a partial differential equation (PDE) with free boundary conditions is induced and American option pricing is the typical problem. In [9,10], and their related references, they showed that the American option

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