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The existence of mild solutions for impulsive fractional partial differential equations

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ABSTRACT

This paper is concerned with the existence of mild solutions for a class of impulsive fractional partial semilinear differential equations. Some errors in Mophou (2010) [2] are corrected, and some previous results are generalized.

1. Introduction

Impulsive fractional differential equations have attracted a considerable interest both in mathematics and applications since Agarwal and Benchohra published the first paper on this topic [1] in 2008; see for example [2–8]. In papers [2,3], the authors studied the existence of the mild solution for some impulsive fractional differential equations. However, in these two papers, there are two problems, (1) the definition of mild solutions given by the authors are not well defined, because classical solutions of the impulsive fractional differential equations do not satisfy the definition of a mild solution given by the authors; (2) the semigroup property $T(t+s) = T(t)T(s)$ for the system is not used correctly.

For example, consider a linear Caputo fractional differential equation

$$(D_*^\alpha y)(t) = -\rho y(t) + f(t), \qquad y(0) = c_1, \quad 0 < \alpha < 1. \tag{1.1}$$

Its classical solution is given by (see [9–11])

$$y(t) = c_1 E_{\alpha,1}(-\rho t^\alpha) + \int_0^t (t-s)^{\alpha-1} E_{\alpha,\alpha}(-\rho(t-s)^\alpha) f(s)\mathrm{d}s,$$

where

$$E_{\alpha,1}(-\rho t^\alpha) = \frac{\rho}{\pi}\sin\pi\alpha \int_0^\infty \mathrm{e}^{-rt}\frac{r^{\alpha-1}}{r^{2\alpha}+2r^\alpha\rho\cos\pi\alpha+\rho^2}\mathrm{d}r,$$

$$t^{\alpha-1}E_{\alpha,\alpha}(-\rho t^\alpha) = -\frac{1}{\pi}\sin\pi\alpha \int_0^\infty \mathrm{e}^{-rt}\frac{r^{\alpha}}{r^{2\alpha}+2r^\alpha\rho\cos\pi\alpha+\rho^2}\mathrm{d}r.$$

Denote $T(t) = t^{\alpha-1}E_{\alpha,\alpha}(-\rho t^\alpha)$, $S(t) = E_{\alpha,1}(-\rho t^\alpha)$. Then $y(t)$ can be expressed as

$$y(t) = c_1 S(t) + \int_0^t T(t-s) f(s)\mathrm{d}s, \tag{1.2}$$

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