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# Entire solutions in monostable reaction–advection–diffusion equations in infinite cylinders☆

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ABSTRACT

This paper is concerned with entire solutions of a monostable reaction–advection–diffusion equation in infinite cylinders without the condition $f'(u) \leq f'(0)$. By constructing a quasi-invariant manifold, we prove that there exist two classes of entire solutions. Furthermore, we show that one class of such entire solutions is unique up to space and time translation.



## 1. Introduction

In this paper, we investigate a nonlinear reaction–advection–diffusion equation in infinite cylinders of the following form

$$\begin{cases} \dfrac{\partial u(t,x,y)}{\partial t} = \Delta u(t,x,y) - q(y)\dfrac{\partial u(t,x,y)}{\partial x} - f(u(t,x,y)), \quad (t,x,y) \in \mathbb{R} \times \Sigma, \\ \dfrac{\partial u(t,x,y)}{\partial \nu} = 0, \quad (t,x,y) \in \mathbb{R} \times \partial\Sigma, \end{cases} \tag{1.1}$$

where $(x, y) \in \Sigma = \mathbb{R} \times \Omega$, $\Omega \subset \mathbb{R}^{n-1}$ is a bounded smooth domain, $\nu$ is the outward unit vector normal to $\partial\Omega$ or $\partial\Sigma$, and $q(y) \in C^{1,\alpha}$ is the advection coefficient with $\alpha \in (0, 1)$ satisfying

$$\int_{\bar{\Omega}} q(y)\mathrm{d}y = 0, \tag{1.2}$$

and the nonlinearity is induced by the function $f$.

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