



# A new composition theorem for square-mean almost automorphic functions and applications to stochastic differential equations

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## ABSTRACT

In this paper, we establish a new composition theorem for square-mean almost automorphic functions under conditions which are different from Lipschitz conditions in the literature. We apply this new composition theorem together with Schauder's fixed point theorem to investigate the existence of square-mean almost automorphic mild solutions for a stochastic differential equation in a real separable Hilbert space. Finally, an interesting corollary is also given for the sub-linear growth cases.

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## 1. Introduction

In this paper, we are mainly focused upon the existence of square-mean almost automorphic mild solutions to the following stochastic differential equations

$$dx(t) = Ax(t)dt + f(t, B_1x(t))dt + g(t, B_2x(t))dW(t), \quad t \in \mathbb{R}, \quad (1.1)$$

where  $A$  generates a compact  $C_0$ -semigroup  $\{T(t)\}_{t \geq 0}$  on a Hilbert space  $L^2(\mathbb{P}, \mathbb{H})$ ,  $B_i$ ,  $i = 1, 2$ , are bounded linear operators, and  $W(t)$  is a two-sided standard one-dimensional Brownian motion defined on the filtered probability space  $(\Omega, \mathcal{F}, \mathbb{P}, \mathcal{F}_t)$ , where  $\mathcal{F}_t = \sigma\{W(u) - W(v); u, v \leq t\}$ . Here  $f$  and  $g$  are appropriate functions to be specified later.

The concept of almost automorphy is an important generalization of the classical almost periodicity. It was introduced in the literature by Bochner [1,2]; for more details about this topic we refer the reader to [3,4]. In recent years, the existence of almost periodic and almost automorphic solutions on different kinds of deterministic differential equations have been considerably investigated in lots of publications [5–14] because of its significance and applications in physics, mechanics and mathematical biology.

Recently, the existence of almost periodic or pseudo-almost periodic solutions to some stochastic differential equations have been considered in many publications such as [15–21] and the references therein. In paper [22], Fu and Liu introduced a new concept of square-mean almost automorphic stochastic processes, and gave some basic properties including the composition theorem. They established the existence and uniqueness of square-mean almost automorphic mild solutions

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