Nonlinear Analysis 74 (2011) 6427–6433

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

# Half-linear ODE and modified Riccati equation: Comparison theorems, integral characterization of principal solution☆

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ARTICLE INFO

*Article history:*
Received 6 December 2010
Accepted 20 June 2011
Communicated by Ravi Agarwal

*MSC:*
34C10

*Keywords:*
Half-linear differential equation
Oscillation criteria
Nonoscillation criteria
Comparison theorems
Riccati equation
Principal solution

ABSTRACT

In this paper, we study the half-linear differential equation with one-dimensional $p$-Laplacian

$$(r(t)\Phi_p(x'))' + c(t)\Phi_p(x) = 0,$$

where $\Phi_p(x) = |x|^{p-2}x$ and $p > 1$. Using a suitable modification of the so-called linearization technique, we derive new results which allow to compare the solutions of two equations with different $p$ and provide new integral characterization of the principal solution.



## 1. Introduction

In this paper, we deal with the half-linear second order differential equation

$$(r(t)\Phi_p(x'))' + c(t)\Phi_p(x) = 0, \tag{1}$$

where $\Phi_p(x) = |x|^{p-2}x$, $p > 1$, and $r$, $c$ are continuous functions, $r(t) > 0$ on the interval $I$, which will be specified below.

The domain of the operator on the left hand side of (1) is defined to be the set of all continuous real-valued functions $x$ defined on $I$ such that $x$ and $r\Phi_p(x')$ are continuously differentiable on $I$.

Eq. (1) attracted big attention as an equation with one-dimensional $p$-Laplacian. It turns out that there is a close relationship between (1) and radially symmetric solutions of PDE with $p$-Laplacian and $(p-1)$-degree power nonlinearity.

Eq. (1) is called half-linear, because a constant multiple of each solution is also a solution. If $p = 2$, then (1) reduces to linear equation, but the linearity is lost in the general case $p \neq 2$. For a comprehensive treatment focused on Eq. (1) and results up to year 2005, see [1].

The main aim of this paper is to prove new comparison results for Eq. (1). In contrast to most other known comparison theorems, we do not compare equations with equal $p$. Another aim of this paper is to provide an alternative integral characterization of the principal solution of a nonoscillatory equation. To achieve these goals we use a modification of the so-called linearization technique; see [2] and the reference therein.

The paper is organized as follows. In Section 2, we recall necessary elements of the oscillation theory for (1), in particular, the Riccati technique, and we derive inequalities which, in turn, are used as a main tool in the proofs of our main results

☆ Supported by the Grant P201/10/1032 of the Czech Science Foundation.
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doi:10.1016/j.na.2011.06.025