Nonlinear Analysis 74 (2011) 5745–5753

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

The existence of three positive solutions of a singular p-Laplacian problem[☆]

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ARTICLE INFO

Article history:
Received 30 January 2011
Accepted 21 May 2011
Communicated by S. Ahmad

Keywords:
Nonlinear elliptic equations
Singular p-Laplacian
Three critical points theorem
$(p-1)$-sublinearity at infinity
Positive solutions

ABSTRACT

We establish the existence of three positive solutions of the p-Laplacian problem, which involves a singular nonlinearity. Three solutions are obtained by using the cutoff argument and the three critical points theorem proved by Ricceri (2000) in [15] and Bonanno (2003) in [12], provided the nonlinear term is a $(p-1)$-sublinear growth at infinity.

1. Introduction

In this paper, we deal with the existence and multiplicity of solutions for the singular nonlinear elliptic boundary value problem:

$$\begin{cases} -\Delta_p u = \lambda a(x) u^{-\gamma} + \lambda f(x,u), & x \in \Omega, \\ u > 0, & x \in \Omega, \\ u = 0, & x \in \partial\Omega, \end{cases} \tag{1.1}$$

where Ω is a bounded smooth open domain of $\mathbb{R}^N$, $\Delta_p u = \operatorname{div}(|\nabla u|^{p-2}\nabla u)$ is the p-Laplacian, $N < p < +\infty$, $\gamma > 0$ is a constant, $\lambda > 0$ is a parameter, f is a Carathéodry function on $\Omega \times [0, +\infty)$, and $a(x) \geq 0$ is a nontrivial measurable function satisfying (H): there are $\varphi_0 \geq 0$ in $C_0^1(\overline{\Omega})$ and $q > N$ such that $a\varphi_0^{-\gamma} \in L^q(\Omega)$.

Note that, in particular, the condition (H) implies that $a \in L^q(\Omega)$. Shi and Yao [1] study the case $f(x,t) = t^\beta$ with $\gamma, \beta \in (0,1)$ and $p = 2$, and obtain one solution by using the super-subsolution methods. Sun et al. [2] obtain two solutions by the Ekeland variational principle. Zhang [3] applies the critical point theory on closed convex sets to obtain two positive solutions, when $f(x,t) \geq 0$ is a general superlinear term. Perera and Silva [4] obtain two solutions of the problem, when $f(x,s)$ is allowed to change sign and is bounded from below by an integrable function on bounded intervals of the variable s (see also [5]).

This paper has four sections. In Section 2, we refer to the books [6–8] for the foundation of this area, and the overview papers [9,1,2,10,3] for the advances and references. The proof of our main result (Theorem 2.6) is based on a recent abstract critical point theorem in Section 3 [11–15]. In Section 4, we give an application of our theorem to the boundary value problem. One can refer to the papers [16,12] for some examples of the three critical points theorem.

☆ Supported by the NSF of China (10971088) and the Fundamental Research Funds for the Central Universities (lzujbky-2010-k10).
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0362-546X/$ – see front matter
doi:10.1016/j.na.2011.05.065