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Existence of solutions of generalized vector equilibrium problems in reflexive Banach spaces

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ABSTRACT

In this paper, a generalized vector equilibrium problem with set-valued maps defined on a reflexive Banach space is considered. By using the recession method, we first give the conditions under which the solution set is non-empty, convex and weakly compact, and then extend it to the strong generalized vector equilibrium problem. This facilitates generalizing and modifying various existence theorems. Furthermore, the topological properties of the solution set are studied and it is shown that the solution set includes some boundary points.

1. Introduction

Let X be a real reflexive Banach space and Y be a real normed linear space. Suppose that 2^Y denotes the family of all subsets of Y, and $C \subseteq Y$ is an order cone, that is a proper, closed and convex cone such that $\operatorname{int} C \neq \emptyset$. Given a non-empty subset $K \subseteq X$ and a set-valued function $F : K \times K \to 2^Y \setminus \emptyset$, the problem of determining the existence of $\bar{x} \in K$ such that

$$F(\bar{x}, y) \cap (-\operatorname{int} C) = \emptyset; \quad \forall y \in K, \tag{1.1}$$

has been extensively studied by many authors in recent years (see [1–8]). This problem may be written as:

$$\text{find } \bar{x} \in K \quad \text{such that } F(\bar{x}, y) \subseteq Y \setminus (-\operatorname{int} C);\ \forall y \in K,$$

which is called generalized vector equilibrium problem (GVEP).

Moreover, the following problem, which is closely related to GVEP (1.1), is named dual generalized vector equilibrium problem (DGVEP):

$$\text{find } \bar{x} \in K \quad \text{such that } F(y, \bar{x}) \cap (\operatorname{int} C) = \emptyset;\ \forall y \in K. \tag{1.2}$$

The solution sets to GVEP and DGVEP are denoted by E_p and E_d, respectively. The strong version of GVEP is considered as follows:

$$\text{find } \bar{x} \in K \quad \text{such that } F(\bar{x}, y) \subseteq C;\ \forall y \in K, \tag{1.3}$$

and its dual form:

$$\text{find } \bar{x} \in K \quad \text{such that } F(y, \bar{x}) \subseteq -C;\ \forall y \in K. \tag{1.4}$$

Problem (1.3) is called the strong generalized vector equilibrium problem (SGVEP) and Problem (1.4) is the dual strong generalized vector equilibrium problem (DSGVEP). The solution sets to SGVEP and DSGVEP are denoted by E_{sp} and E_{sd},

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