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On the asymptotic stability of discontinuous systems analysed via the averaging method

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ABSTRACT

The averaging method is one of the most powerful methods used to analyse differential equations appearing in the study of nonlinear problems. The idea behind the averaging method is to replace the original equation by an averaged equation with simple structure and close solutions. A large number of practical problems lead to differential equations with discontinuous right-hand sides. In a rigorous theory of such systems, developed by Filippov, solutions of a differential equation with discontinuous right-hand side are regarded as being solutions to a special differential inclusion with upper semi-continuous right-hand side. The averaging method was studied for such inclusions by many authors using different and rather restrictive conditions on the regularity of the averaged inclusion. In this paper we prove natural extensions of Bogolyubov's first theorem and the Samoilenko–Stanzhitskii theorem to differential inclusions with an upper semi-continuous right-hand side. We prove that the solution set of the original differential inclusion is contained in a neighbourhood of the solution set of the averaged one. The extension of Bogolyubov's theorem concerns finite time intervals, while the extension of the Samoilenko–Stanzhitskii theorem deals with solutions defined on the infinite interval. The averaged inclusion is defined as a special upper limit and no additional condition on its regularity is required.

1. Introduction

The averaging method is one of the methods most used for analysing differential equations of the form

$$\dot{x} = \epsilon f(t, x), \tag{1}$$

appearing in the study of nonlinear problems. The idea behind the averaging method is to replace the original equation by the averaged one:

$$\dot{x} = \epsilon \bar{f}(x) = \epsilon \lim_{T\to\infty} \frac{1}{T} \int_0^T f(t, x)\mathrm{d}t. \tag{2}$$

This equation is simpler and has solutions close to the solutions of the original equation. A rigorous justification of the method is given by Bogolyubov's first theorem containing an estimate for the distance between the solutions of the exact

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