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Systems of first order impulsive functional differential equations with deviating arguments and nonlinear boundary conditions

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ABSTRACT

This paper is concerned with the existence of solutions for systems of first order impulsive functional differential equations with deviating arguments and nonlinear boundary conditions. By establishing new comparison results and applying the monotone iterative technique, we obtain the sufficient conditions for the existence of extremal system of solutions.

1. Introduction

Impulsive differential equations arise naturally from a wide variety of applications, such as control theory, physics, chemistry, population dynamics, biotechnology, industrial robotics, optimal control, etc. [1–9]. Therefore, it is very important to develop a general theory for differential equations with impulses including some basic aspects of this theory.

The method of upper and lower solutions coupled with the monotone iterative technique has been applied successfully to obtain existence of solutions for nonlinear differential equations in recent years (see [10–19]). In the above mentioned papers, the main results are formulated and proved under the assumption of existing upper and lower solutions in the usual order. Recently, many authors investigated existence results for the non-ordered case [20–25]. Nieto and Rodriguez-Lopez [16,17] consider periodic boundary value problem for the following first order functional differential equations

$$\begin{cases} u'(t) = f(t, u(t), u(\alpha(t))), & t \in J, \\ u(0) = u(T), \end{cases}$$

where $t \in J = [0, T]$ $(T > 0), f \in C(J \times R \times R, R), \alpha \in C(J, R), 0 \le \alpha(t) \le t$.

In 2009, Wang et al. [24] considered boundary value problems for functional differential equations with nonlinear boundary conditions under the assumption of existing upper and lower solutions in the reverse order

$$\begin{cases} u'(t) = f(t, u(t), u(\alpha(t))), & t \in J, \\ h(u(0)) = u(T), \end{cases}$$

where $t \in J = [0, T]$ $(T > 0), f \in C(J \times R \times R, R), h \in C^1(R, R), h(0) \le 0, \alpha \in C(J, J)$.

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