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# The asymptotic behavior of the ground state solutions for biharmonic equations☆

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ABSTRACT

In this paper we study the asymptotic behavior of the ground state solutions of the Hénon type biharmonic equation $\Delta^2 u = |x|^\alpha u^{p-1}$ in $\Omega$, $u > 0$ in $\Omega$ and $u = \frac{\partial u}{\partial \mathbf{n}} = 0$ on $\partial\Omega$, where $\Omega$ is the unit ball in $\mathbb{R}^N$, $\alpha > 0$, $p > 2$. We prove that the ground state solution $u_p$ concentrates on a boundary point and has a unique maximum point as $p \to 2^* = \frac{2N}{N-4}$, which deduce that the ground state solution $u_p$ is not radially symmetric.

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## 1. Introduction

In this paper we consider the Dirichlet problem

$$\begin{cases} \Delta^2 u = |x|^\alpha u^{p-1} & \text{in } \Omega, \\ u > 0 & \text{in } \Omega, \\ u = \dfrac{\partial u}{\partial \mathbf{n}} = 0 & \text{on } \partial\Omega, \end{cases} \tag{1.1}$$

where $\Omega$ is the unit ball in $\mathbb{R}^N (N > 4)$, $\mathbf{n}$ is the unit exterior normal on $\partial\Omega$, $\alpha > 0$ and $2 < p < 2^* = \frac{2N}{N-4}$, where $2^*$ is the critical Sobolev exponent. We are interested in the asymptotic behavior of the ground state solutions of (1.1). For $\alpha \geq 0$, $2 < p < 2^*$, define

$$S_{\alpha,p} := \inf \left\{ \frac{\int_\Omega |\Delta u|^2}{\left(\int_\Omega |x|^\alpha |u|^p\right)^{2/p}},\ u \in H_0^2(\Omega) \setminus \{0\} \right\}. \tag{1.2}$$

Since the embeddings $H_0^2(\Omega) \hookrightarrow L^p(\Omega)$ are compact for $1 \leq p < 2^*$, there exists a nontrivial function $u_p \in H_0^2(\Omega)$ such that $S_{\alpha,p}$ is achieved by $u_p$. Up to sign and normalization $u_p$ is a positive ground state solution of (1.1). Our main results are as follows.

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