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Cauchy problem for fast diffusion equation with localized reaction☆

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ABSTRACT

This paper studies the Cauchy problem for the fast diffusion equation with a localized reaction. We establish the Fujita type theorem to the problem, and then obtain the diffusion-independent blow-up rate for the non-global solutions. Moreover, we prove that the blow-up set for the problem consists of a single point under large initial data. These conclusions are quite different from those for the slow diffusion case.

1. Introduction

In this paper, we consider the Cauchy problem of fast diffusion equation with localized reaction

$$\begin{cases} u_t = (u^m)_{xx} + a(x)u^p, & (x,t) \in \mathbb{R} \times (0,T), \\ u(x,0) = u_0(x) \geq 0, & x \in \mathbb{R}, \end{cases} \tag{1.1}$$

where $0 < m < 1$, $p > 0$, $u_0 \in L^1(\mathbb{R}) \cap L^\infty(\mathbb{R}) \cap C(\mathbb{R})$, $a(x)$ is smooth, nonnegative, and compactly supported. For simplicity, suppose $\operatorname{supp} a(x) = [-L, L]$, namely, $c_1\chi_{[-L_1,L_1]} \leq a(x) \leq c_2\chi_{[-L,L]}$ with $L_1 = L/2$ and $c_2 > c_1 > 0$.

Recently, the slow diffusion case of (1.1) with $m > 1$ was studied by Ferreira et al. in [1]. The semilinear case of $m = 1$ was considered by Pinsky [2].

The Eq. (1.1) could be considered as a special case of the more general problem

$$\begin{cases} u_t = \operatorname{div}(|\nabla u|^{l-1}\nabla(u^m)) + g(x)\,(t^s+1)u^p, & (x,t) \in \mathbb{R}^n \times (0,T), \\ u(x,0) = u_0(x), & x \in \mathbb{R}^n, \end{cases} \tag{1.2}$$

with $g(x) \geq 0$, $g(x) \sim |x|^\sigma$ as $|x| \to \infty$. If $\sigma = s = 0$, $l = m = 1$, then (1.2) is reduced to the classical heat equation considered by Fujita [3]. The case of $s = 0$ (or $\sigma = 0$), $l = m = 1$ was studied in [2,4–6]. When $l = 1$, the critical Fujita exponent of (1.2) was established in [7] (for $s \geq 0$, $\sigma > -\min\{n, 2\}$) and [1] (for $s = 0$, $\sigma = -\infty$). The case of $m = 1$ with $\frac{n-1}{n+1} < l < 1$, $s \geq 0$, $\sigma > n(1-l) - (1+l+2s)$ was treated in [8]. Under some other assumption on l, m, s, σ, the

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