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Infinitely many periodic solutions for second-order (p, q)-Laplacian differential systems[☆]

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ABSTRACT

Some existence theorems are obtained for infinitely many periodic solutions for second-order (p, q)-Laplacian differential systems by using a general variational principle due to B. Ricceri.

1. Introduction

In this paper, we consider the following second-order (p, q)-Laplacian differential system

$$\begin{cases} -\left(|u_1'|^{p-2}u_1'\right)' + \sigma_1(t)|u_1|^{p-2}u_1 = \nabla_{u_1}F(t, u_1, u_2), \\ -\left(|u_2'|^{q-2}u_2'\right)' + \sigma_2(t)|u_2|^{q-2}u_2 = \nabla_{u_2}F(t, u_1, u_2), \quad \text{a.e. } t \in [0, \omega], \\ u_1(0) - u_1(\omega) = u_1'(0) - u_1'(\omega) = 0, \\ u_2(0) - u_2(\omega) = u_2'(0) - u_2'(\omega) = 0, \end{cases} \tag{1}$$

where $1 < p, q < +\infty, \omega > 0, \sigma_1, \sigma_2 : [0, \omega] \to (0, +\infty)$ are continuous, and $F : [0, \omega] \times \mathbf{R}^N \times \mathbf{R}^N \to \mathbf{R}$ satisfies the following assumption:

- $F(t, x_1, x_2)$ is measurable in t for each $(x_1, x_2) \in \mathbf{R}^N \times \mathbf{R}^N$;
- $F(t, x_1, x_2)$ is continuously differentiable in (x_1, x_2) for a.e. $t \in [0, \omega]$;
- there exist $a_1, a_2 \in C(\mathbf{R}^+, \mathbf{R}^+)$ and $b \in L^1(0, \omega; \mathbf{R}^+)$ such that

$$\max\{|F(t, x_1, x_2)|, |\nabla_{x_1}F(t, x_1, x_2)|, |\nabla_{x_2}F(t, x_1, x_2)|\} \le [a_1(|x_1|) + a_2(|x_2|)]b(t)$$

for all $(x_1, x_2) \in \mathbf{R}^N \times \mathbf{R}^N$ and a.e. $t \in [0, \omega]$.

In the last twenty years, many scholars studied the following second-order Hamiltonian system with periodic boundary conditions

$$\begin{cases} -u''(t) = \nabla F(t, u(t)), \quad \text{a.e. } t \in [0, \omega], \\ u(0) - u(\omega) = u'(0) - u'(\omega) = 0. \end{cases} \tag{2}$$

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