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The monotone iterative method and zeros of Bessel functions for nonlinear singular derivative dependent BVP in the presence of upper and lower solutions

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ABSTRACT

In this paper we consider a class of nonlinear singular boundary value problems

$$-\left(x^{\alpha}y'(x)\right)'+x^{\alpha}f\left(x,y(x),x^{\alpha}y'(x)\right)=0,\quad 0<x<1,\ y'(0)=y'(1)=0,$$

for $\alpha \geq 1$. We assume that the source function $f(x, y, x^{\alpha}y')$ is Lipschitz in $x^{\alpha}y'$ and one-sided Lipschitz in y. The initial approximations are an upper solution $u_0(x)$ and a lower solution $v_0(x)$ which can be ordered in one way, $v_0(x) \leq u_0(x)$, or the other, $u_0(x) \leq v_0(x)$. We propose an iterative scheme and establish the existence of solutions bounded by v_0 and u_0, and allow $\partial f/\partial y$ to take both positive and negative values. The method is constructive in nature and can be used to generate solutions of the nonlinear singular boundary value problems.

1. Introduction

The upper and lower solution technique is the most promising technique as far as singular boundary value problems are concerned [1]. Initially the shooting method [2] and nonlinear alternative methods [3] were used for treating singular nonlinear problems, though originally both methods were developed for treating non-singular problems. Nonlinear alternative methods can be further divided into topological degree theory ones [4] and topological transversality ones [5].

Nowadays, different techniques are being coupled with the upper and lower solution technique, e.g., those of the topological degree theory [6], topological transversality [7,8], the monotone iterative method [9–14] and quasilinearization [15].

In the present work we have utilized the upper and lower solution technique related to the monotone iterative method where the upper and lower solutions are well-ordered and where they are not well-ordered. For the non-singular case, i.e., $-y''+f(x,y,y')=0$, Cherpion et al. [9] considered the following iterative scheme:

$$-y''_{n+1}+\lambda y_{n+1}=-f(x,y_n,y'_n)+\lambda y_n,\qquad y'_{n+1}(0)=y'_{n+1}(1)=0,\tag{1.1}$$

and proved that it is possible to make a good choice of λ such that the approximations converge monotonically to solutions of (1.1) for both well-ordered and non-well-ordered upper and lower solutions. They utilized the properties of $\cos\sqrt{|\lambda|}x(\sin\sqrt{|\lambda|}x)$ and $\cosh\sqrt{\lambda}x(\sinh\sqrt{|\lambda|}x)$ to make a good choice of λ.

Chawla and Shivkumar [10] considered the following class of nonlinear singular boundary value problems:

$$-(x^{\alpha}y'(x))'=x^{\alpha}f(x,y),\qquad 0<x<1,\ y'(0)=0,\ y'(1)=1\tag{1.2}$$

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