Nonlinear Analysis 74 (2011) 2062–2070

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Fixed points and stability in linear neutral differential equations with variable delays

Abdelouaheb Ardjouni *, Ahcene Djoudi

University of Annaba, Department of Mathematics, P.O. Box 12, Annaba 23000, Algeria

ARTICLE INFO

Article history:
Received 21 September 2010
Accepted 22 October 2010

MSC:
34K20
34K30
34K40

Keywords:
Fixed points
Stability
Neutral differential equation
Integral equation
Variable delays

ABSTRACT

In this paper we consider the asymptotic stability of a generalized linear neutral differential equation with variable delays by using the fixed point theory. An asymptotic stability theorem with a necessary and sufficient condition is proved, which improves and generalizes some results due to Burton (2003) [3], Zhang (2005) [14], Raffoul (2004) [13], and Jin and Luo (2008) [12]. Two examples are also given to illustrate our results.

1. Introduction

Certainly, the Lyapunov direct method has been successfully used to investigate the stability properties of a wide variety of ordinary, functional and partial differential equations. Nevertheless, the application of this method to problems of stability in differential equations with delay has encountered serious difficulties if the delay is unbounded or if the equation has unbounded terms [1–3]. Recently, investigators such as Burton, Zhang, Raffoul, Jin, Luo and others have noticed that some of these difficulties vanish or might be overcome by means of fixed point theory (see [1–14]). The fixed point theory does not only solve the problem on stability but has a significant advantage over Lyapunov's direct method. The conditions of the former are often averages but those of the latter are usually pointwise (see [1]).

Let $a, b, c, b_j, c_j \in C\left(\mathbb{R}^+, \mathbb{R}\right)$, and $\tau, \tau_j \in C\left(\mathbb{R}^+, \mathbb{R}^+\right)$ with $t - \tau(t) \to \infty$ and $t - \tau_j(t) \to \infty$ as $t \to \infty$. Here $C\left(S_1, S_2\right)$ denotes the set of all continuous functions $\varphi : S_1 \to S_2$ with the supremum norm $\|.\|$.

In [3], Burton studied the equation

$$x'(t) = -b(t)x\left(t - \tau(t)\right), \tag{1.1}$$

and proved the following theorem.

Theorem A (*Burton [3]*). *Suppose that $\tau(t) = r$ and there exists a constant $\alpha < 1$ such that*

$$\int_{t-r}^{t} |b(s+r)|\, ds + \int_0^t |b(s+r)|\, e^{-\int_s^t b(u+r)du} \left(\int_{s-r}^{s} |b(u+r)|\, du\right) ds \le \alpha, \tag{1.2}$$

* Corresponding author.
E-mail addresses: abd_ardjouni@yahoo.fr (A. Ardjouni), adjoudi@yahoo.com (A. Djoudi).

0362-546X/$ – see front matter
doi:10.1016/j.na.2010.10.050