Nonlinear Analysis 74 (2011) 3775–3785

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Positive solutions for second order impulsive differential equations involving Stieltjes integral conditions

Tadeusz Jankowski *

Gdansk University of Technology, Department of Differential Equations and Applied Mathematics, 11/12 G.Narutowicz Str., 80–233 Gdańsk, Poland

ARTICLE INFO

Article history:
Received 4 February 2011
Accepted 14 March 2011
Communicated by: Ravi Agarwal

MSC:
34B10

Keywords:
Impulsive second order differential equations
Boundary conditions including a Stieltjes integral
Positive solutions

ABSTRACT

In this paper, the existence of at least three positive solutions to impulsive second order differential equations is investigated. Sufficient conditions which guarantee the existence of positive solutions are obtained, by using the Avery–Peterson theorem. An example is added to illustrate the results.

1. Introduction

For $J = [0, 1]$, let $0 = t_0 < t_1 < \cdots < t_m < t_{m+1} = 1$. Put $J' = (0, 1) \setminus \{t_1, t_2, \ldots, t_m\}$. Put $\mathbb{R}_+ = [0, \infty)$ and $J_k = (t_k, t_{k+1}]$, $k = 0, 1, \ldots, m-1$, $J_m = (t_m, t_{m+1})$.

Let us consider second order impulsive differential equations of type

$$\begin{cases} x''(t) + \alpha(t) f(t, x(t)) = 0, & t \in J', \\ \Delta x'(t_k) = Q_k(x(t_k)), & k = 1, 2, \ldots, m, \\ x(0) = 0, \qquad x(1) = \lambda[x], \end{cases} \tag{1}$$

where as usual $\Delta x'(t_k) = x'(t_k^+) - x'(t_k^-)$; $x'(t_k^+)$ and $x'(t_k^-)$ denote the right and left limits of x' at t_k, respectively. Here $\lambda[u]$ denotes a linear functional on $C(J)$ given by

$$\lambda[u] = \int_0^1 u(t) \mathrm{d}\Lambda(t)$$

involving a Stieltjes integral with a suitable function Λ of bounded variation.

Impulsive differential equations are discussed, for example in books [1,2], see also [3]. The theory of impulsive differential equations has become an important aspect of differential equations. There are many papers in which the problem of existence of positive solutions to differential equations with boundary conditions have been discussed. However, only a few papers have appeared where such problem was discussed to impulsive differential equations with boundary conditions, see for example, [4,3,5–11].

* Fax: +48 58 3472821.
E-mail address: tjank@mifgate.pg.gda.pl.

0362-546X/$ – see front matter
doi:10.1016/j.na.2011.03.022