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Renormings and the fixed point property in non-commutative L_1-spaces

Carlos A. Hernández-Linares, Maria A. Japón *

Departamento de Análisis Matemático, Facultad de Matemáticas, Universidad de Sevilla, Tarfia s/n, 41012, Sevilla, Spain

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ABSTRACT

Let $\mathcal{M}$ be a finite von Neumann algebra. It is known that $L_1(\mathcal{M})$ and every non-reflexive subspace of $L_1(\mathcal{M})$ fail to have the fixed point property for non-expansive mappings (FPP). We prove a new fixed point theorem for this class of mappings in non-commutative $L_1(\mathcal{M})$ Banach spaces which lets us obtain a sufficient condition such that a closed subspace of $L_1(\mathcal{M})$ can be renormed to satisfy the FPP. As a consequence, we deduce that the predual of every atomic finite von Neumann algebra can be renormed with the FPP.

1. Introduction

Let X be a Banach space. It is said that X has the fixed point property (FPP) if every non-expansive mapping defined from a closed convex bounded subset into itself has a fixed point. The existence of fixed points for non-expansive mappings has been widely studied and it is well-known that the geometry of the Banach space plays a fundamental role in guaranteeing that there are fixed points for non-expansive mappings. For instance, if X is either uniformly convex or X is reflexive with normal structure, X satisfies the FPP, and classical non-reflexive Banach spaces, such as c_0 or ℓ_1, fail to have the FPP (for more background results the reader can consult [1] or [2] and the references therein). Many interesting problems are still open concerning the fixed point property and important results have been discovered in the last few years that have motivated new lines of research. For instance, Lin [3] has proved that there exists an equivalent norm $|||\cdot|||$ in the sequence space ℓ_1 such that $(\ell_1, |||\cdot|||)$ has the FPP. Lin's renorming answered, in a negative way, the long open question of whether FPP implies reflexivity. What is more, Lin's result connects renorming theory with fixed point theory in the following way: if a Banach space X fails the FPP with its original norm, can X be renormed to have this property? In this line, Domínguez Benavides [4] gave a positive answer in the case where the Banach space is reflexive. However, for non-reflexive Banach spaces we do not know a general solution. For instance, in [5] the authors proved that every renorming of ℓ_∞, $\ell_1(\Gamma)$ and $c_0(\Gamma)$, with Γ an uncountable set, contains an asymptotically isometric copy of either ℓ_1 or c_0 and, therefore, that these spaces cannot be renormed to have the FPP. In [6] the authors extend Lin's techniques in order to find new non-reflexive (and non-isomorphic to ℓ_1) Banach spaces failing the FPP that can be renormed to have this property and, using the convergence in measure topology, they give a sufficient condition such that a closed subspace of the measure space $L_1[0, 1]$ can also be renormed with the FPP. In this paper we focus on non-commutative L_1-spaces associated with finite von Neumann algebras.

* Corresponding author.
E-mail addresses: carloshl@us.es (C.A. Hernández-Linares), japon@us.es (M.A. Japón).

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