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Characterization of solutions having finite Morse index for some nonlinear PDE with supercritical growth

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ABSTRACT

In this paper, we study solutions of the equation $-\Delta u = (u^+)^p$ in $\Omega \subset \mathbb{R}^N$, where $\frac{N+2}{N-2} \leq p < p_c(N)$; see (3) for the definition of $p_c(N)$. We first classify all solutions (not necessarily bounded) having finite Morse index for $\Omega = \mathbb{R}^N$ or $\Omega = \mathbb{R}^N_+$ with Dirichlet boundary condition. When Ω is a regular bounded domain of $\mathbb{R}^N$, for a family of solutions with Dirichlet boundary condition (u_n), we prove that $\|u_n\|_{L^\infty(\Omega)}$ is bounded if and only if the Morse index sequence, $i(u_n)$, of u_n is bounded. Finally, we prove the same result for other nonlinearities $f(x, u)$ which have similar growth respectively at $\pm\infty$.

1. Introduction and main results

We are concerned here with solutions of

$$\text{(I)} \quad \begin{cases} -\Delta u = f(x, u(x)) & \text{in } \Omega \\ u = 0 & \text{on } \partial\Omega \end{cases}$$

where Ω is a bounded, open, smooth domain in $\mathbb{R}^N$, $N \geq 3$; $f(x, t)$ is continuous on $\overline{\Omega} \times \mathbb{R}$, differentiable with respect to t, and $\frac{\partial f}{\partial t}(x, t)$ is continuous on $\overline{\Omega} \times \mathbb{R}$. We study here the supercritical case (see hypothesis (H_1) below).

When f has a subcritical growth, Bahri and Lions [1], Yang [2], Harrabi et al. [3] and Harrabi et al. [4] prove that bounds on solutions are equivalent to bounds on their Morse indices. If we come back to the work of Bahri and Lions [5] and Bahri [6], we see that these a priori bounds are required for topological methods to establish existence and multiplicity results of (I). In some sense, the L^∞-bounds which are known for positive solutions, see [7,8], are somewhat related to this result.

In [1] the proof of these bounds are reduced to show the following Liouville-type theorem: there is no non-trivial bounded solution of $-\Delta u = |u|^{p-1}u$ in $\mathbb{R}^N$ or $\mathbb{R}^N_+$ with finite Morse index and $1 < p < \frac{N+2}{N-2}$. Results of this kind are also obtained by de Figueiredo and Yang [9], Ramos and Rodrigues [10] for the bilaplacian, Angenent and Van Der Vorst [11], Ramos and Yang [12] for an elliptic system. For example in [9,13,14,10,15], the authors use the a priori estimates for a sequence of solutions having bounded Morse index to obtain existence results when the Palais–Smale compactness condition is not satisfied (see Section 3, p. 606 in [15]). In fact, after blow-up, they prove some Liouville-type results for the limiting solutions with finite Morse index.

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