Nonlinear Analysis 74 (2011) 5015–5022

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

The asymptotically additive topological pressure on the irregular set for asymptotically additive potentials

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ARTICLE INFO

Article history:
Received 24 March 2011
Accepted 28 April 2011
Communicated by S. Ahmad

MSC:
37D35
37A05

Keywords:
Variational principle
Topological pressure
Multifractal analysis

ABSTRACT

Given a continuous dynamical system (X, T) with the specification property, and a sequence of asymptotically additive continuous functions, we consider the irregular set for it and show that this set is either empty or carries full asymptotically additive topological pressure.

1. Introduction

Let (X, T) be a topological dynamical system (TDS for short) in the sense that X is a compact metric space with the metric d and $T : X \to X$ is a continuous map. For a continuous function $f : X \to \mathbb{R}$, the following irregular set for f is extensively studied

$$\widehat{X}_f := \left\{ x \in X : \lim_{n\to\infty} \frac{1}{n} \sum_{i=0}^{n-1} f(T^i x) \text{ does not exist} \right\}.$$

This set arises naturally in the context of multifractal analysis, and the following decomposition of the space X holds

$$X = \bigcup_{\alpha\in\mathbb{R}} X_{f,\alpha} \cup \widehat{X}_f$$

where $X_{f,\alpha}$ denotes the set of points for which the Birkhoff average of f is equal to α.

The theory of multifractal analysis is a subfield of the dimension theory of dynamical systems; its main purpose is to study the complexity of the level sets of invariant local quantities obtained from a given dynamical system. See the books [1,2] for details about the theory of multifractal analysis.

Using topological entropy as the dimension characteristic, Takens and Verbitskiy have obtained multifractal analysis results for the class of maps with specification [3,4], however, they did not consider the irregular set. Pesin and Pitskel [5] first studied the irregular set and showed that it carried the full entropy in the case of the Bernoulli shift on two symbols. Later, Barreira and Schmeling [6] showed that the irregular set of a generic Hölder continuous function on a conformal repeller has full entropy by using symbolic dynamics. In [7], Chen et al. proved that the irregular set of continuous function

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doi:10.1016/j.na.2011.04.065