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Some properties of geodesic semi-E-convex functions

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1. Introduction

Convexity and generalized convexity play an important and significant role in optimization theory. In order to extend the validity of their results to larger classes of optimization, these concepts have been generalized and extended in several directions using novel and innovative techniques. The concepts of *E*-convex sets and *E*-convex functions were introduced by Youness [1], which have some important applications in various branches of mathematical sciences [2–4]. The initial results of Youness [1] inspired a great deal of subsequent work which has greatly expanded the role of *E*-convexity in optimization theory; see [5–9]. Syau and Lee [8] introduced the concept of *E*-quasiconvex functions and discussed some properties of *E*-convex and *E*-quasiconvex functions. Agarwal et al. [10] introduced a different notion of functions named as semistrictly geodesic η -preinvex functions on Riemannian manifolds. Jimenez et al. [11] studied the dual problem and established weak, strong, and convex duality results.

Rapcsak [12] and Udriste [13] proposed a generalization of convexity on Riemannian manifold. In this setting, the linear space is replaced by a Riemannian manifold and the line segment by a geodesic. Iqbal et al. [14] generalized convexity and introduced geodesic *E*-convex sets and geodesic *E*-convex functions on Riemannian manifolds.

Motivated by earlier research works, we introduce a new class of functions, which are called geodesic semi-*E*-convex functions and discuss some of their properties. We also define geodesic semi-*E*-quasiconvex functions and geodesic semi-*E*-pseudoconvex functions, and present some results about these functions.

2. Preliminaries

In this section, we recall some notations, definitions and basic properties of Riemannian manifolds, which will be used throughout the paper. For the standard material on differential geometry, one can consult [13].

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ABSTRACT

In this paper, we introduce a new class of functions called geodesic semi-*E*-convex functions and discuss some of their properties.

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