Nonlinear Analysis 74 (2011) 4496–4502

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

A study of nonlinear problems for the p-Laplacian in $\mathbb{R}^n$ via Ricceri's principle☆

Said El Manouni *

Al-Imam University, Faculty of Sciences, Department of Mathematics, P.O. Box 90950, Riyadh 11623, Saudi Arabia

ARTICLE INFO

Article history:
Received 27 December 2010
Accepted 4 April 2011
Communicated by S. Carl

MSC:
35J35
35J60
35J66

Keywords:
p-Laplacian
Multiple nontrivial solutions
Ricceri's principle
Regularity result

ABSTRACT

In this paper, we consider the following nonlinear eigenvalue problems for the p-Laplacian:

$$-\operatorname{div}\left(a(x)|\nabla u|^{p-2}\nabla u\right) = \lambda f(x,u) + \mu g(x,u) \quad \text{in } \mathbb{R}^n$$
$$\lambda,\mu > 0, \qquad \lim_{|x|\to\infty} u = 0,$$

where $1 < p < n$, $\lambda,\mu > 0$, a is a measurable bounded function, and f and g are nonlinearities having subcritical growth with respect to u. We prove multiple nontrivial solutions using a recent principle of Ricceri (2009) [10]. A regularity result is also established.

1. Introduction

In this work, we study the following nonlinear problem:

$$\begin{cases} -\operatorname{div}(a(x)|\nabla u|^{p-2}\nabla u) = \lambda f(x,u) + \mu g(x,u) & \text{in } \mathbb{R}^n \\ \lambda,\mu > 0, \qquad \lim\limits_{|x|\to\infty} u = 0, \end{cases} \tag{1.1}$$

where $1 < p < n$, $\lambda,\mu > 0$ are parameters, $f(x,t)$ and $g(x,t)$ are two functions having subcritical growth with respect to t, and a is a measurable function such that $a \in L^\infty(\mathbb{R}^n)$ with $\operatorname{essinf} a > 0$. More precisely, we assume that f is a Carathéodory function satisfying the following condition:

$$|f(x,t)| \le m(x)|t|^\gamma \quad \forall x \in \mathbb{R}^n \text{and } \forall t \in \mathbb{R}, \tag{1.2}$$

where m is a positive function such that $m \in L^{\frac{p^*}{p^*-1}}(\mathbb{R}^n) \cap L^{\frac{\nu}{\nu-1}\left(\frac{p^*}{p^*-(\gamma+1)}\right)}(\mathbb{R}^n)$, with $p < \gamma + 1 < \nu < p^*$, where p^* denotes the critical Sobolev exponent, i.e., $p^* = \frac{np}{n-p}$.

Regarding the function $g = g(x,t)$, it is assumed to be a measurable function (it may be a higher-order term) with respect to x in $\mathbb{R}^n$ for every t in $\mathbb{R}$, and it is continuous with respect to t in $\mathbb{R}$ for almost every x in $\mathbb{R}^n$ such that there exists a positive function h satisfying

$$\sup_{(x,t)\in\mathbb{R}^n\times\mathbb{R}\setminus\{0\}} \frac{|g(x,t)|}{h(x)|t|^r} < +\infty, \tag{1.3}$$

☆ This work was supported by the Deanery of Scientific Research at Al-Imam University under project No. 290101202, Riyadh, KSA.

* Tel.: +966 560921788.
E-mail addresses: samanouni@imamu.edu.sa, manouni@hotmail.com.

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.04.013