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On the fractional differential equations with uncertainty

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ABSTRACT

This paper is based on the concept of fuzzy differential equations of fractional order introduced by Agarwal et al. [R.P. Agarwal, V. Lakshmikantham, J.J. Nieto, On the concept of solution for fractional differential equations with uncertainty, Nonlinear Anal. 72 (2010) 2859–2862]. Using this concept, we prove some results on the existence and uniqueness of solutions of fuzzy fractional differential equations.

1. Introduction

Fractional calculus is a generalization of differentiation and integration to an arbitrary order. First work, devoted exclusively to the subject of fractional calculus, is the book by Oldham and Spanier [1]. A rigorous study of fractional calculus can be found in [2]. In recent years, fractional differential equations have attracted a considerable interest both in mathematics and in applications. Many recently developed models in areas like rheology, viscoelasticity, electrochemistry, diffusion processes, etc. are formulated in terms of fractional derivatives or fractional integrals. The books [3,4] present the first introduction to fractional differential equations and their applications, but a complete study of fractional differential equations can be found in [5]. One of the most recent works on the subject of fractional calculus is the book by Lakshmikantham et al. [6]. Some theoretical aspects on the existence and uniqueness results for fractional differential equations have been considered by some authors recently [7–25]. A differential and integral calculus for fuzzy-valued mappings was developed in papers of Dubois and Prade [15–17], Puri and Ralescu [26,27] and Kaleva [28]. The book [29] presents the first introduction to fuzzy differential equations and their applications. In paper [12], the authors make some interesting observations about Peano's Theorem for fuzzy differential equations. Also, for some recent results and applications in fuzzy differential equations theory, see [11,30–33].

This paper is based on the concept of fuzzy fractional differential equations which is introduced by Agarwal et al. [34]. The paper is organized as follows. In Section 2 we recall some basic knowledge of fuzzy calculus and fractional calculus. Several basic concepts and properties of fuzzy fractional calculus are presented in Section 3. In Section 4 we prove some results on the existence and uniqueness of solutions of fuzzy fractional differential equations.

2. Definitions and preliminary results

Denote by E the space of fuzzy sets in $\mathbb{R}$ with the following properties:

(i) u is normal, that is, there exists an $x_0 \in \mathbb{R}$ such that $u(x_0) = 1$;

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