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Uniform energy decay rates for nonlinear viscoelastic wave equation with nonlocal boundary damping☆

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ABSTRACT

In this paper, we consider the uniform decay estimates of solutions for the viscoelastic wave equation

$$u_{tt} - \kappa_0 \Delta u + \int_0^t g(t-s)\mathrm{div}\,[a(x)\nabla u(s)]\,\mathrm{d}s + b(x)h(u_t) = 0 \quad \text{in } \Omega \times (0,\infty).$$

Under weak assumptions on the functions g, h and f, we prove the energy functional decays exponentially or polynomially to zero as the time goes to infinity by introducing brief Lyapunov functions and precise priori estimates.

1. Introduction

In this paper, we are concerned with the energy decay rate of the problem

$$\begin{cases} u_{tt} - \kappa_0 \Delta u + \displaystyle\int_0^t g(t-s)\mathrm{div}[a(x)\nabla u(s)]\mathrm{d}s + b(x)h(u_t) = 0, \quad (x,t) \in \Omega \times (0,\infty), \\ -\dfrac{\partial u}{\partial \nu} + \displaystyle\int_0^t g(t-s)(a(x)\nabla u(s)) \cdot \nu \mathrm{d}s = f(u), \quad (x,t) \in \Sigma_0 = \Gamma_0 \times (0,\infty), \\ u(x,t) = 0, \quad (x,t) \in \Sigma_1 = \Gamma_1 \times (0,\infty), \\ u(x,0) = u_0(x), \qquad u_t(x,0) = u_1(x), \quad x \in \Omega, \end{cases} \tag{1.1}$$

where κ_0 is a positive constant, Ω is a bounded domain of $R^n (n \geq 1)$ with a smooth boundary $\Gamma := \partial\Omega$ with $\Gamma = \Gamma_0 \cup \Gamma_1$, $\Gamma_0 \cap \Gamma_1 = \emptyset$ and Γ_0, Γ_1 have positive measures.

In recent years, uniform decay rates for the solutions to wave equation with memory terms have attracted attention of many mathematicians. For example, Santos [1] and Cavalcanti et al. [2] proved the uniform decay estimates of solutions of Eq. (1.1) with $a(x) = 1$, $b(x) = 0$ and Dirichlet boundary conditions and proved uniform decay estimates of solutions to Eq. (1.1) with Dirichlet boundary conditions. In the same direction, Muñoz Rivera et al. [3,4] established the uniform decay of solutions provided that the function $b(x) = 0$ in Ω. Later Cavalcanti et al. [5] studied the uniform decay estimates of

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