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On the distribution of zeros of solutions of first order delay differential equations

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ABSTRACT

This paper contains new estimates for the distance between adjacent zeros of solutions of the first order delay differential equation

$$x'(t) + p(t)x(t-\tau) = 0$$

where p is a nonnegative continuous function. Our results are new and improve some known criteria. Illustrative examples are given to show the strength of some obtained results over the known ones.

1. Introduction

The oscillation theory of functional differential equations has been the subject of intensive investigations by an increasing number of mathematicians. This activity reflects the importance of this topic for applications in many fields such as mathematical biology and neural networks (see [1–3] for more applications and details). The question of estimating the distance between adjacent zeros of all solutions of first order equations is one of the main challenges in this theory. Many authors have been attracted by this problem, for example, the equation

$$x'(t) + p(t)x(t-\tau) = 0, \tag{1}$$

has been studied by [4–8] (see also the monographs [9,10]) while its variable delayed version has been investigated by [11].

The neutral generalization of (1), namely,

$$[x(t) - cx(t-\sigma)]' + p(t)x(t-\tau) = 0,$$

has been studied by [12–14] while [15,16] studied its variable delayed form. It is also worth noting that first order differential equations with one or two delays have been considered by [17]. Also a first order integrodifferential equation has been studied by [18].

The papers [4,5] contain results of fundamental type as they studied the behavior of the number of zeros $N(t)$ and the number of sign changes $v(t)$ of all solutions of (1) on any interval $[t-\tau,\ t]$ for all $t \geq 0$ only under the assumption that $p(t)$ is of constant sign. The other works [6–8] and [4, Theorem 5] are all interested in obtaining an upper bound of the distance between adjacent zeros of all solutions of (1) under more restrictions on $p(t)$.

In this work, we prove some new fundamental results about the distribution of zeros of all solutions of Eq. (1). We also find new estimates for the distance between adjacent zeros of all solutions of (1). We use different techniques than those

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