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Existence of solutions for generalized variational inequality problems in Banach spaces

Prapairat Junlouchai, Somyot Plubtieng *

Department of Mathematics, Faculty of Science, Naresuan University, Phitsanulok 65000, Thailand

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ABSTRACT

In this paper, we prove the existence of solutions of generalized variational inequality for upper semicontinuous multivalued mappings with compact contractible values over compact convex subsets in a reflexive Banach space with a Fréchet differentiable norm. Moreover, we give some conditions that guarantee the existence of solutions of generalized variational inequality for upper semicontinuous multivalued mappings with compact contractible values over unbounded closed convex subsets. The result obtained in this paper improves and extends the recent ones announced by Yu and Yang [J. Yu, H. Yang, Existence of solutions for generalized variational inequality problems, Nonlinear Anal., 71 (2009) e2327–e2330] and many others.

1. Introduction

Let E be a reflexive and smooth Banach space, K a closed convex subset in E. Let $f : K \to E^*$ be a mapping. The variational inequality problem, denoted by $VI(K, f)$, is to find a vector $x^* \in K$ such that

$$\langle f(x^*), x - x^*\rangle \geq 0 \quad \text{for all } x \in K.$$

If $T : K \to 2^{E^*}$ is a set-valued mapping, then we have the generalized variational inequality problem, denoted by $GVI(K, T)$, which is to find a vector $x^* \in K$ such that there exists a vector $u^* \in T(x^*)$ satisfying

$$\langle u^*, y - x^*\rangle \geq 0 \quad \text{for all } y \in K.$$

Variational inequality theory with applications is an important part of nonlinear analysis. It has been applied intensively to different fields such as mechanics, game theory, economics, optimization theory and nonlinear programming. Since 1960s, researchers have obtained many existence results of solutions for variational inequality problems and nonlinear complementarity problems; see [1–9]. In 1966, Hartman and Stampacchia published their celebrated existence result for problem $VI(K, f)$ asserting that $VI(K, f)$ possesses a solution, given that f is continuous, E is a finite-dimensional Euclidean space, and K is a nonempty compact convex set [1]. Since then, many extensions of their result have been obtained. See e.g. [3] for the corresponding result in a reflexive Banach space assuming the monotonicity of operators.

Recently, the notion of exceptional family of elements was introduced to deal with the solvability of variational inequality problems over unbounded sets; see [10] and the references therein. In [10], Han et al. obtained some significant results on the solvability of variational inequality problems over unbounded sets. Specifically, they presented a new concept of

* Corresponding author. Tel.: +66 55261000; fax: +66 55261025.
E-mail address: Somyotp@nu.ac.th (S. Plubtieng).

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