Nonlinear Analysis 74 (2011) 5697–5704

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Rearrangements and minimization of the principal eigenvalue of a nonlinear Steklov problem

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ARTICLE INFO

Article history:
Received 26 March 2011
Accepted 17 May 2011
Communicated by Ravi Agarwal

MSC:
35J20
35P30
49R50

Keywords:
Minimization
Principal eigenvalue
Steklov eigenvalue problem
Rearrangement theory
Existence
Uniqueness

ABSTRACT

This paper, motivated by Del Pezzo et al. (2006) [1], discusses the minimization of the principal eigenvalue of a nonlinear boundary value problem. In the literature, this type of problem is called Steklov eigenvalue problem. The minimization is implemented with respect to a weight function. The admissible set is a class of rearrangements generated by a bounded function. We merely assume the generator is non-negative in contrast to [1], where the authors consider weights which are positively away from zero, in addition to being two-valued. Under this generality, more physical situations can be modeled. Finally, using rearrangement theory developed by Geoffrey Burton, we are able to prove uniqueness of the optimal solution when the domain of interest is a ball.

1. Introduction

This paper is motivated by [1]. In [1], the authors investigate a minimization problem related to the following Steklov eigenvalue problem:

$$\begin{cases} -\Delta_p u + |u|^{p-2}u + \alpha\chi_E|u|^{p-2}u = 0 & \text{in } D,\\ |\nabla u|^{p-2}\dfrac{\partial u}{\partial \nu} = \lambda|u|^{p-2}u & \text{on } \partial D, \end{cases} \tag{1.1}$$

where D is a smooth bounded domain in $\mathbb{R}^n$. Here, Δ_p denotes the usual p-Laplace operator; that is, $\Delta_p u = \nabla\cdot(|\nabla u|^{p-2}\nabla u)$, χ_E is the characteristic function of $E \subseteq D$, and α is a positive parameter. Denoting the principal eigenvalue of (1.1) by $\lambda_1(E)$, the minimization problem considered in [1] is

$$\inf_{E\subseteq D,\ |E|=A} \lambda_1(E), \tag{1.2}$$

where $A \in [0, |D|]$. Henceforth, for a measurable set F, $|F|$ denotes the n-dimensional Lebesgue measure of F.

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doi:10.1016/j.na.2011.05.056