Nonlinear Analysis 74 (2011) 235–243

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

On the eigenvalues of weighted $p(x)$-Laplacian on $\mathbb{R}^N$

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ARTICLE INFO

Article history:
Received 15 July 2009
Accepted 19 August 2010

MSC:
35D05
35J60
46E30
47J10

Keywords:
$p(x)$-Laplacian operator
Variable exponent Lebesgue–Sobolev space
Eigenvalue
Electrorheological fluids

ABSTRACT

This paper, following the theory of partial differential equations on variable exponent Sobolev spaces, is mainly concerned with the $p(x)$-Laplacian eigenvalue problem with a weight function on $\mathbb{R}^N$. The results show that the spectrum of such problems contains a continuous family of eigenvalues.

1. Introduction

Electrorheological fluids are colloidal suspensions of a certain type, consisting of dielectric particles dispersed in an insulating oil. The marvelous feature of such fluids is that they can solidify into a jelly-like state almost instantaneously when subjected to an externally applied electric field with moderate strength, with a stiffness varying proportionally to the field strength. The liquid–solid transformation is reversible. Once the applied field is removed, the original flow state is recovered. This phenomenon is known as the Winslow effect.

Electrorheological fluids have the quality and potential for a wide field of applications. These include for example robotics, aircraft and aerospace applications. We refer the reader to [1–3] for more information.

There exist several possibilities for modeling the physics of electrorheological fluids. In [2], Rajagopal and Růžička have developed a model which takes into account the complex interaction of electromagnetic fields and the moving liquid. The constitutive equation for the motion of an electrorheological fluid is given by

$$\frac{\partial}{\partial t}u + \operatorname{div} S(u) + (u.\nabla)u + \nabla\pi = f \tag{1.1}$$

where $u : \mathbb{R}^{3+1} \to \mathbb{R}^3$ is the velocity of the fluid at a point, $\nabla = (\partial_1, \partial_2, \partial_3)$ is the gradient operator, π is the pressure, f represents external forces and the stress tensor S is of the form

$$S(u)(x) = \mu(x)(1 + |\mathrm{D}u(x)|^2)^{\frac{p-2}{2}}\mathrm{D}u(x)$$

where $p = p(x)$ and $\mathrm{D}u = \frac{1}{2}(\nabla u + \nabla u^T)$ is the symmetric part of the gradient of u. Rajagopal and Růžička established an existence result for problem (1.1) in variable exponent spaces.

Variable exponent Lebesgue spaces $L^{p(x)}$ appeared for the first time in a paper of Orlicz [4] in 1931. When $p(.)$ is finite almost everywhere these spaces are called Orlicz–Musielak spaces [5]. If $p(x) \equiv p$ is constant, $L^{p(x)}$ coincides with the classical Lebesgue space L^p and the norms in the two spaces are equal.

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0362-546X/$ – see front matter
doi:10.1016/j.na.2010.08.037