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Hopf bifurcation of a predator–prey system with stage structure and harvesting[☆]

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ABSTRACT

In this paper, a two-species predator–prey system with stage structure and harvesting is investigated. The existence of Hopf bifurcations of the system is given. And the stability and directions of Hopf bifurcations are determined by applying the normal form theory and the center manifold theorem.

1. Introduction

Linear and nonlinear delayed differential equations have been extensively applied in many areas such as biology, networks, robot engineering, control of signals and so on. For the theory of the above models, based on functional differential equations, one can refer to [1–3] and so on. In the paper, we will consider a kind of nonlinear delayed differential equations which describes a two-species predator–prey system with stage structure and harvesting. Similar models have been studied in [4–6] and so on. However, the stage structure of species has been ignored in much of the literature. In fact, there are many species whose individual members have two stages: immature and mature (see [7–12]). In these models, the age to maturity is represented by a time delay, leading to systems of retarded functional differential equations. For general models of population growth one can see [13,14].

In this paper, we consider the nonlinear delayed differential equations

$$\begin{cases} \dfrac{\mathrm{d}u_{1i}(t)}{\mathrm{d}t} = \alpha_0 u_1(t) - \gamma u_{1i}(t) - \alpha u_1(t-\tau), \\ \dfrac{\mathrm{d}u_1(t)}{\mathrm{d}t} = \alpha u_1(t-\tau) - \beta u_1^2(t) - a_1 u_1(t) u_2(t) - h u_1(t), \\ \dfrac{\mathrm{d}u_2(t)}{\mathrm{d}t} = u_2(t)(-r_1 + a_2 u_1(t) - b u_2(t)), \\ u_{1i}(0) > 0, \qquad u_2(0) > 0, \qquad u_1(t) = \psi(t) \geq 0, \quad -\tau \leq t \leq 0, \end{cases} \tag{1.1}$$

where $u_{1i}(t),\ u_1(t)$ represent respectively the immature and mature prey population densities; $u_2(t)$ represents the density of the predator population; $a_1 > 0$ is the transformation coefficient of the mature predator population; α represents the immatures who were born at time $t-\tau$ and survive to time t (with the immature death rate γ), and τ represents the

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