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Nonlinear systems with singular vector ϕ-Laplacian under the Hartman-type condition

Qihuai Liu [a,b], Dingbian Qian [a,*], Bin Chu [a]

[a] School of Mathematical Sciences, Soochow University, Suzhou 215006, PR China

[b] School of Mathematics and Computing Science, Guilin University of Electronic Technology, Guilin 541002, PR China

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ABSTRACT

Based on a continuation principle, we give some existence results of solutions for the nonlinear problem

$$\big(\phi(u')\big)' = f(t,u),$$
$$u(0) = u(T), \qquad u'(0) = u'(T)$$

involving singular vector ϕ-Laplacian with $\phi : B(a) \to \mathbb{R}^N$ $(0 < a \leq +\infty)$ or $\phi : \mathbb{R}^N \to B(a)$ $(0 < a < +\infty)$ a homeomorphism under the Hartman-type condition or the modified Hartman-type condition.

1. Introduction

Boundary value problems including singular or bounded ϕ-Laplacians

$$\big(\phi(u')\big)' = h(t,u,u'), \qquad \ell(u,u') = 0$$

have received a lot of attention with respect to the existence and multiplicity of solutions, where $\phi : B(a) \to \mathbb{R}^N$ $(0 < a \leq +\infty)$ or $\phi : \mathbb{R}^N \to B(a)$ $(0 < a < +\infty)$ is a homeomorphism involving singular vector ϕ-Laplacian, $f : [0,T] \times \mathbb{R}^N \to \mathbb{R}^N$ is continuous and $\ell(u,u') = 0$ denotes the various boundary conditions on $[0,T]$.

Such problems arise from the relativistic oscillator models and curvature operator equations [1]. For example, the special relativity operator $\phi : B(1) \to \mathbb{R}^N$ is given by

$$\phi(u) = \frac{u}{\sqrt{1-|u|^2}}$$

and a curvature operator $\phi : \mathbb{R}^N \to B(1)$ is given by

$$\phi(u) = \frac{u}{\sqrt{1+|u|^2}},$$

where $B(a)$ is the open ball of center 0 and radius a.

Since 2004, with a number of papers, Bereanu and Mawhin have studied such problems with periodic, Neumann or Dirichlet conditions. See, for example, [2–10] and the references therein. In these papers, the various boundary value

* Corresponding author. Tel.: +86 051267165225; fax: +86 051267165225.
E-mail addresses: qhuailiu@gmail.com (Q. Liu), dbqian@suda.edu.cn (D. Qian).

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