Nonlinear Analysis 74 (2011) 4746–4753

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

Global existence and blow-up for a weakly dissipative μDP equation

Martin Kohlmann

Institute for Applied Mathematics, University of Hannover, D-30167 Hannover, Germany

ARTICLE INFO

Article history:
Received 21 February 2011
Accepted 21 April 2011
Communicated by Enzo Mitidieri

MSC:
35Q35
35G25
58D05

Keywords:
μ-Degasperis–Procesi equation
Weak dissipation
Diffeomorphism group of the circle
Blow-up
Global existence

ABSTRACT

In this paper, we study a weakly dissipative variant of the periodic Degasperis–Procesi equation. We show the local well-posedness of the associated Cauchy problem in $H^s(\mathbb{S})$, $s > 3/2$, and discuss the precise blow-up scenario for $s = 3$. We also present explicit examples for globally existing solutions and blow-up.

1. Introduction

In recent years, the family

$$y_t = -(y_x u + b u_x y), \quad y = u - u_{xx}, \tag{1}$$

of nonlinear equations has been studied extensively; see [1,2]. Here, $u(t, x)$ depends on a time variable $t \geq 0$ and a space variable x with $x \in \mathbb{S} \simeq \mathbb{R}/\mathbb{Z}$ for the periodic equation and $x \in \mathbb{R}$ in the non-periodic case. Eq. (1) is called b-equation and (1) becomes the Camassa–Holm (CH) equation for $b = 2$ and the Degasperis–Procesi (DP) equation if $b = 3$. Moreover, the corresponding family of μ-equations, where

$$y = \mu(u) - u_{xx}, \quad \mu(u) = \int u(t, x)\mathrm{d}x$$

in (1), has been studied, e.g., in [3,4]. It is known that the b-equation models the unidirectional motion of 1D water waves over a flat bed; for the hydrodynamical relevance we refer to, e.g., [5–9]. It turned out that the b-equation is integrable only if $b \in \{2, 3\}$ and in [4], the authors mention that a similar result is conjectured for the family of μ-equations. The Cauchy problems for the b-equation and its μ-variant have been discussed in [2,4]. In particular, for $b \in \{2, 3\}$, local well-posedness results in the periodic and in the real line-case as well as blow-up and criteria for the global existence of strong and weak solutions have been established; see, e.g., [10–12]. In addition, CH and DP and μCH and μDP admit peaked solitons, which make them attractive among the integrable equations; cf. [5,6,4].

In general, it is difficult to avoid energy dissipation mechanisms in the modeling of fluids. Ott and Sudan [13] discussed the KdV equation under the influence of energy dissipation. Ghidaglia [14] studied the behavior of solutions of the weakly

E-mail address: kohlmann@ifam.uni-hannover.de.

0362-546X/$ – see front matter © 2011 Elsevier Ltd. All rights reserved.
doi:10.1016/j.na.2011.04.043