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# Coupled fixed points in partially ordered metric spaces and application

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ABSTRACT

In this paper, we prove some coupled fixed point theorems for mappings having a mixed monotone property in partially ordered metric spaces. The main results of this paper are generalizations of the main results of Bhaskar and Lakshmikantham [T. Gnana Bhaskar, V. Lakshmikantham, Fixed point theorems in partially ordered metric spaces and applications, Nonlinear Anal. TMA 65 (2006) 1379–1393]. As an application, we discuss the existence and uniqueness for a solution of a nonlinear integral equation.



## 1. Introduction and preliminaries

The existence of a fixed point for contraction type mappings in partially ordered metric spaces has been considered recently by Ran and Reurings [1], Bhaskar and Lakshmikantham [2], Nieto and Lopez [3,4], Agarwal et al. [5], Lakshmikantham and Ćirić [6] and Samet [7]. The existence of solutions for matrix equations or ordinary differential equations by applying fixed point theorems are presented in [8,2,9,3,4,1].

In [2], Bhaskar and Lakshmikantham introduced notions of a mixed monotone mapping and a coupled fixed point and proved some coupled fixed point theorems for mixed monotone mapping and discussed the existence and uniqueness of a solution for a periodic boundary value problem.

**Definition 1.1** (*[2]*). Let $(X, \leq)$ be a partially ordered set and $F : X \times X \to X$. The mapping $F$ is said to have the mixed monotone property if $F(x, y)$ is monotone non-decreasing in $x$ and is monotone non-increasing in $y$, that is, for any $x, y \in X$,

$$x_1, x_2 \in X, \quad x_1 \leq x_2 \Rightarrow F(x_1, y) \leq F(x_2, y)$$

and

$$y_1, y_2 \in X, \quad y_1 \leq y_2 \Rightarrow F(x, y_1) \geq F(x, y_2).$$

**Definition 1.2** (*[2]*). An element $(x, y) \in X \times X$ is called a coupled fixed point of the mapping $F : X \times X \to X$ if

$$x = F(x, y) \quad \text{and} \quad y = F(y, x).$$

The main theoretical results of Bhaskar and Lakshmikantham in [2] are the following coupled fixed point theorems.

**Theorem 1.3** (*[2]*). *Let $(X, \leq)$ be a partially ordered set and suppose there exists a metric $d$ on $X$ such that $(X, d)$ is a complete metric space. Let $F : X \times X \to X$ be a continuous mapping having the mixed monotone property on $X$. Assume that there exists*

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