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Existence of entire positive solutions for a critical system of nonlinear elliptic equations

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ABSTRACT

In this paper, using the fibering method introduced by Pohozaev, we establish an existence of multiple nontrivial positive solutions for a system of nonlinear elliptic equations in $\mathbb{R}^N$ with lack of compactness studying the properties of Palais–Smale sequence of the energy functional associated with the system.

1. Introduction

In this paper, we show the existence of nonnegative weak solutions of the following system of nonlinear elliptic equations

$$\begin{cases} -\Delta_p u = u^{\alpha} v^{\beta+1} & \text{in } \mathbb{R}^N, \\ -\Delta_q v = u^{\alpha+1} v^{\beta} & \text{in } \mathbb{R}^N, \\ u, v \geq 0, \end{cases} \tag{1}$$

under the following critical condition:

$$\frac{\alpha+1}{p^*} + \frac{\beta+1}{q^*} = 1.$$

Here, $N \geq 3$, $1 < p, q < N$, $\alpha, \beta > -1$ and $p^* := \frac{pN}{N-p}$, $q^* := \frac{qN}{N-q}$ designate respectively the effective critical exponents relating to the Sobolev embedding $W^{1,p}(\mathbb{R}^N) \subset L^r(\mathbb{R}^N)$ and $W^{1,q}(\mathbb{R}^N) \subset L^r(\mathbb{R}^N)$. Let us mention that in the case where $p = q$ and $\alpha + \beta + 2 = p^*$, the scalar case ($u = v$) of (1) leads to the standard problem

$$-\Delta_p u = u^{p^*-1} \quad \text{in } \mathbb{R}^N,$$

with well-known papers and abundant literature (one cite for example of classical references [1–5]). On the other hand, the study on existence results of entire solutions for a system like (1) is not new. Such investigations have been done in [6] for more general nonlinear gradient functionals. In that paper, existence and nonexistence results have been obtained assuming

$$\frac{\alpha+1}{p^*} + \frac{\beta+1}{q^*} < 1 < \frac{\alpha+1}{p} + \frac{\beta+1}{q}.$$

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