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# Nonlinear Analysis



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## A Nash type solution for hemivariational inequality systems

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#### 1. Introduction

### ABSTRACT

In this paper, we prove an existence result for a general class of hemivariational inequality systems using the Ky Fan version of the KKM theorem Fan (1984) [10] or Tarafdar fixed points Tarafdar (1987) [11]. As application, we give an infinite-dimensional version for the existence result of Nash generalized derivative points introduced recently by Kristály (2010) [5]. We also give an application to a general hemivariational inequality system. © 2011 Elsevier Ltd. All rights reserved.

In the last few years, many papers have been dedicated to the study of the existence and multiplicity of solutions for hemivariational inequality systems or differential inclusion systems defined on bounded or unbounded domains; see [1–6]. In these papers, the authors use the critical point theory for locally Lipschitz functions, combined with the *Principle of Symmetric Criticality* and different topological methods. For a comprehensive treatment of hemivariational inequality and hemivariational inequality systems on bounded domains using the critical point theory for non-smooth functionals, we refer the reader to the monographs of Motreanu and Rădulescu [7] and Motreanu and Panagiotopoulos [8]. For very recent results concerning variational inequalities and elliptic systems using the critical point theory and different variational methods, see also the book by Kristály et al. [9].

The aim of this paper is to prove the existence of at least one solution for a general class of hemivariational inequality systems on a closed and convex set (either bounded or unbounded), without using the critical point theory. We apply a version of the well-known theorem of Knaster–Kuratowski–Mazurkiewicz due to Ky Fan [10] or the Tarafdar fixed point theorem [11]. We start the paper by giving in Section 2 the assumptions and by formulating the hemivariational inequality system problem that we study. The main results concerning the existence of at least one solution for the hemivariational inequality systems that we study are given in Section 3. Section 4 contains applications to Nash and Nash generalized derivative points and existence results for some abstract class of hemivariational inequality systems.

#### 2. Assumptions and formulation of the problem

Let  $X_1, X_2, \ldots, X_n$  be reflexive Banach spaces and  $Y_1, Y_2, \ldots, Y_n, Z_1, \ldots, Z_n$  Banach spaces, such that there exist linear operators  $T_i : X_i \to Y_i, T_i : X_i \to Z_i$  for  $i \in \{1, \ldots, n\}$ . We suppose that the following condition holds:

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