Nonlinear Analysis 74 (2011) 1761–1768

Contents lists available at ScienceDirect

Nonlinear Analysis

journal homepage: www.elsevier.com/locate/na

An implementable augmented Lagrange method for solving fixed point problems with coupled constraints☆

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ARTICLE INFO

Article history:
Received 3 February 2010
Accepted 25 October 2010

MSC:
37C25
65K99

Keywords:
Fixed point problem
Coupled constraints
Augmented Lagrange function
Semismooth function
Newton method

ABSTRACT

An augmented Lagrange function method for solving fixed point problems with coupled constraints is studied, and a theorem of its global convergence is demonstrated. The semismooth Newton method is used to solve the inner problems for obtaining approximate solutions, and numerical results are reported to verify the effectiveness of the augmented Lagrange function method for solving three examples with more than 1000 variables.

1. Introduction

Consider the problem for computing a fixed point of an extremal mapping with coupled constraints: find $v^* \in \Omega_0$ such that

$$v^* \in \arg\min\{\Phi(v^*, \omega) \mid g(v^*, \omega) \leq 0, \omega \in \Omega_0\}, \tag{1.1}$$

where $\Phi : \Re^n \times \Re^n \to \Re, g : \Re^n \times \Re^n \to \Re^m$ are mappings and $\Omega_0 \subseteq \Re^n$ is a convex closed set. It is assumed that $\Phi(v, \omega)$ and each component of the vector-function $g(v, \omega)$ are convex in $\omega \in \Omega_0$ for any $v \in \Omega_0$. It is also assumed that the extremal mapping $\omega(v) \equiv \arg\min\{\Phi(v, \omega) \mid g(v, \omega) \leq 0, \omega \in \Omega_0\}$ is defined for all $v \in \Omega_0$ and the solution set $\Omega^* = \{v^* \in \Omega | v^* \in \omega(v^*)\} \subset \Omega_0$ of the initial problem is non-empty. The non-emptiness of Ω^* comes from the continuity of $\Phi(v, \omega)$, and the convexity of $\Phi(v, \omega)$ in ω for any $v \in \Omega_0$; see for instance [1].

Problems with coupled constraints arise in many fields of mathematics. Among these are economic equilibrium models with budget constraints [2], n-person games [3], equilibrium programming problems and hierarchical programming problems [4].

The equilibrium programming problem with coupled constraints (1.1) is studied in [5], where properties of symmetric and skew-symmetric functions are discussed, and the differential feedback control gradient-type method is suggested and its global convergence proved. However, only theoretical results are presented in [5]; no numerical results are reported.

In this paper we study an implementable augmented Lagrange method for solving problem (1.1) and report numerical results of the proposed method for solving three problems with more than 1000 variables.

☆ The research is supported by the National Natural Science Foundation of China under project No. 11071029 and the Fundamental Research Funds for the Central Universities.

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0362-546X/$ – see front matter
doi:10.1016/j.na.2010.10.048