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Nodal solutions of boundary value problems with boundary conditions involving Riemann–Stieltjes integrals

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ABSTRACT

We study the nonlinear boundary value problem consisting of the equation $-y'' = \sum_{i=1}^{m} w_i(t)f_i(y)$ and a boundary condition involving a Riemann–Stieltjes integral. By relating it to the eigenvalues of the corresponding linear Sturm–Liouville problem with a two-point separated boundary condition, we obtain results on the existence and nonexistence of nodal solutions of this problem. The shooting method and an energy function are used to prove the main results.

1. Introduction

We are concerned with the boundary value problem (BVP) consisting of the equation

$$-y'' = \sum_{i=1}^{m} w_i(t)f_i(y), \quad t \in (a, b), \tag{1.1}$$

and the boundary condition (BC)

$$\begin{aligned} &\cos\alpha\, y(a) - \sin\alpha\, y'(a) = 0, \quad \alpha \in [0, \pi), \\ &y(b) - \int_a^b y(s)\mathrm{d}\xi(s) = 0, \end{aligned} \tag{1.2}$$

where $m \geq 1$ is an integer, $a, b \in \mathbb{R}$ with $a < b$ and the integral in BC (1.2) is the Riemann–Stieltjes integral with respect to $\xi(s)$ with $\xi(s)$ a function of bounded variation. In the case where $\xi(s) = s$, the Riemann–Stieltjes integral in the second condition of (1.2) reduces to the Riemann integral. In the case that $\xi(s) = \sum_{j=1}^{d} k_j\chi(s - \eta_j)$, where $d \geq 1$, $k_j \in \mathbb{R}, j = 1, \ldots, m$, $\{\eta_j\}_{j=1}^{d}$ is a strictly increasing sequence of distinct points in (a, b), and $\chi(s)$ is the characteristic function on $[0, \infty)$, i.e.,

$$\chi(s) = \begin{cases} 1, & s \geq 0, \\ 0, & s < 0, \end{cases}$$

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