



# On Stepanov-like (pseudo) almost automorphic functions<sup>☆</sup>

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## ABSTRACT

We prove new composition theorems for Stepanov-like almost automorphic and Stepanov-like pseudo-almost automorphic functions. An application is given to a class of evolution equations with Stepanov-like pseudo-almost automorphic perturbations.

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## 1. Introduction

The concept of almost automorphic functions, which was introduced by Bochner [1], is an important generalization of the classical almost periodic functions. Stimulated by the work [1], many interesting generalizations of the almost automorphic functions, such as, Stepanov-like almost automorphic functions, pseudo-almost automorphic functions, have been introduced and investigated in recent years (see, e.g., [2–16] and references therein and the definitions of these functions will be given in Section 2). Meanwhile, the applications of the new theory for these functions to various types of semilinear differential equations were also studied extensively (see, e.g., [2–16] and references therein). This note is concerned with the composition of Stepanov-like almost automorphic and Stepanov-like pseudo-almost automorphic functions. As one has known, this is a key property of such functions for many further studies in theory and applications.

In [10], N'Guérékata proved the following composition theorem for almost automorphic functions.

**Theorem A.** Let  $\varphi : \mathbb{R} \rightarrow X$  be almost automorphic and  $f : \mathbb{R} \times X \rightarrow X$  be almost automorphic in  $t$  for each  $x \in X$ : moreover,  $f$  satisfies

(H0) there exists a constant  $L > 0$  such that for all  $u, v \in X$  and  $t \in \mathbb{R}$ ,

$$\|f(t, u) - f(t, v)\| \leq L\|u - v\|.$$

Then  $f(\cdot, \varphi(\cdot)) : \mathbb{R} \rightarrow X$  is almost automorphic.

In [3], Blot et al. obtained the following composition theorem for almost automorphic functions.

**Theorem B.** Let  $(X, d)$  be a complete metric space,  $\varphi : \mathbb{R} \rightarrow X$  be almost automorphic and  $f \in AAU(\mathbb{R} \times X, X)$ , i.e.,  $f$  satisfies the following two conditions.

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