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Positive solutions for a mixed and singular quasilinear problem

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ABSTRACT

We deal with the existence of solution for the nonlinear elliptic problem

$$\begin{cases} -\Delta_p u = \lambda f(x, u) & \text{in } \Omega, \\ u > 0 \quad \text{in } \Omega, \quad u(x) = 0 & \text{on } \partial\Omega, \end{cases}$$

where Ω denotes a smooth bounded domain in $\mathbb{R}^N$, $\Delta_p u := div(|\nabla u|^{p-2}\nabla u)$ with $1 < p < N$, is the p-Laplacian operator, $f : \Omega \times (0, \infty) \to [0, \infty)$ is a suitable function and $\lambda > 0$ is a real parameter. The nonlinearity f is allowed to behave like either $f(x, s) \overset{s\to 0}{\longrightarrow} \infty$ and/or $f(x, s) \overset{s\to\infty}{\longrightarrow} \infty$ for each $x \in \Omega$.

1. Introduction

We shall establish the results on the existence of solutions for the nonlinear elliptic problem

$$\begin{cases} -\Delta_p u = \lambda f(x, u) & \text{in } \Omega, \\ u > 0 \quad \text{in } \Omega, \qquad u(x) = 0 & \text{on } \partial\Omega, \end{cases} \tag{1}$$

where Ω denotes a smooth bounded domain in $\mathbb{R}^N$, $\Delta_p u := \mathrm{div}(|\nabla u|^{p-2}\nabla u)$, $1 < p < N$, is the usual p-Laplacian operator, $f : \Omega \times (0, \infty) \to [0, \infty)$ is a continuous function and $\lambda > 0$ is a real parameter.

The principal fact in this paper is that the nonlinearity f is allowed to behave like either $f(x, s) \overset{s\to 0}{\longrightarrow} \infty$ or $f(x, s) \overset{s\to\infty}{\longrightarrow} \infty$. That is, f can be either singular-super-linear at 0 or super-linear at infinity.

It is well known that such singular elliptic problems arise in contexts of chemical heterogeneous catalysts, non-Newtonian fluids and also the theory of heat conduction in electrically conducting materials; see [1–4] for a detailed discussion.

These kinds of problems have been studied extensively in the past years for separable nonlinearities $f(x, t) = b(x)g(t)$, where b and g are appropriate functions.

Lazer and McKenna in [5], motivated by pioneering work of Crandall et al. [6], study the problem

$$\begin{cases} -\Delta u = b(x)u^{-\gamma} & \text{in } \Omega, \\ u(x) = 0 & \text{on } \partial\Omega, \end{cases} \tag{2}$$

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