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Life span of solutions for a semilinear heat equation with initial data having positive limit inferior at infinity

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ABSTRACT

We present a new upper bound of the life span of positive solutions of a semilinear heat equation for initial data having positive limit inferior at space infinity. The upper bound is expressed by the data in limit inferior, not in every direction, but around a specific direction. It is also shown that the minimal time blow-up occurs when initial data attains its maximum at space infinity.

1. Introduction

We consider the life span of positive solutions of the Cauchy problem for a semilinear heat equation

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u + F(u), & (x,t) \in \mathbb{R}^n \times (0,\infty), \\ u(x,0) = \phi(x) \geq 0, & x \in \mathbb{R}^n, \end{cases} \tag{1}$$

where $n \geq 1$, and initial data ϕ is a bounded continuous function on $\mathbb{R}^n$. Throughout this paper, we assume that $F(u)$ satisfies

$$F(u) \geq u^p \quad \text{for } u \geq 0, \tag{2}$$

with $p > 1$.

There exists a large amount of literature on the blow-up problem for the equation

$$\begin{cases} \dfrac{\partial u}{\partial t} = \Delta u + u^p, & (x,t) \in \mathbb{R}^n \times (0,\infty), \\ u(x,0) = \phi(x) \geq 0, & x \in \mathbb{R}^n. \end{cases} \tag{3}$$

Results in [1–6] are summarized as follows:

(i) Let $p \in (1, 1+2/n]$. Then, every nontrivial solution of Eq. (3) blows up in finite time.
(ii) Let $p \in (1+2/n, \infty)$. Then Eq. (3) has a time-global classical solution for some initial data ϕ.

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