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Time-periodic solutions of a nonlinear wave equation

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ABSTRACT

The existence of a time-periodic solution of an n-dimensional nonlinear wave equation is established with $n = 2$ and 3.

1. Introduction

Consider the problem

$$w'' - \Delta w + |w|^{p-2} w = f \quad \text{in } Q = \Omega \times (0, T), \qquad (1.1)$$
$$w = 0 \quad \text{on } \partial\Omega \times (0, T), \qquad \{w, w'\}|_{t=0} = \{w, w'\}|_{t=T} \quad \text{in } \Omega,$$

where Ω is a bounded open subset of R^n, $n = 2, 3$, with a smooth boundary. The forcing term f is in a closed subset of $N^{\perp}$ and

$$N^{\perp} = \left\{ g : \int_0^T (g, \varphi) dt = 0 \; \forall \varphi \in L^2(Q), \varphi(., 0) = \varphi(., T), \int_0^T \varphi dt = 0 \right\}.$$

The existence of a solution of (1.1) is established in Section 3 when $2 \le p \le 4$ and $\Omega \subset R^3$ and for planar domains with $2 \le p < \infty$. For the linear wave equation we shall study time-periodic solutions of the linear wave equation with a forcing term dependent on the unknown.

Let $\{\alpha, \beta\}$ be in $\{H_0^1(\Omega) \cap L^p(\Omega)\} \times L^2(\Omega)$, and consider the initial boundary problem

$$w'' - \Delta w + |w|^{p-2} w = f \quad \text{in } Q,$$
$$w = 0 \quad \text{on } \partial\Omega \times (0, T), \qquad \{w, w'\}|_{t=0} = \{\alpha, \beta\} \quad \text{in } \Omega. \qquad (1.2)$$

In Section 4, we study the optimization problem

$$A = \left\{ \|\alpha - w(., T)\|_{H_0^1(\Omega)} + \|\beta - w'(., T)\| : \forall \{\alpha, \beta, w\} \text{ solution of (1.2)}, \forall \{\alpha, \beta\} \in H_0^1(\Omega) \times L^2(\Omega) \right\}. \qquad (1.3)$$

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