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On the uniqueness of weak solutions for the 3D viscous magneto-hydrodynamic equations

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ABSTRACT

A uniqueness result of weak solutions for the 3D viscous magneto-hydrodynamic equations is proved by means of the Fourier localization technique and Bony's paraproduct decomposition.

1. Introduction

In this paper, we consider the 3D incompressible magneto-hydrodynamic (MHD) equations

$$\text{(MHD)}\quad \begin{cases} \dfrac{\partial u}{\partial t} - \nu \Delta u + u \cdot \nabla u = -\nabla p - \dfrac{1}{2}\nabla b^2 + b \cdot \nabla b, \\ \dfrac{\partial b}{\partial t} - \eta \Delta b + u \cdot \nabla b = b \cdot \nabla u, \\ \nabla \cdot u = \nabla \cdot b = 0, \\ u(0,x) = u_0(x), \quad b(0,x) = b_0(x). \end{cases} \tag{1.1}$$

Here u, b describe the flow velocity vector and the magnetic field vector respectively, p is a scalar pressure, $\nu > 0$ is the kinematic viscosity, $\eta > 0$ is the magnetic diffusivity, while u_0 and b_0 are the given initial velocity and initial magnetic field with $\nabla \cdot u_0 = \nabla \cdot b_0 = 0$. If $\nu = \eta = 0$, (1.1) is called the ideal MHD equations.

As same as the 3D Navier–Stokes equations, the regularity and uniqueness of weak solutions for the 3D MHD equations remains open [1]. For the 3D Navier–Stokes equations, the Serrin-type criterion states that the Leray–Hopf weak solutions are unique and regular provided the following conditions hold:

$$L^q(0,T;L^r) \quad \text{with } \frac{2}{q} + \frac{3}{r} = 1,\ 3 \le r \le \infty,\ [2\text{–}5]$$

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