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Population models with diffusion, strong Allee effect, and nonlinear boundary conditions

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ABSTRACT

We consider a population model with diffusion, a strong Allee effect per capita growth function, and constant yield harvesting. In particular, we focus our study on a population living in a patch, $\Omega \subseteq \mathbb{R}^n$ with $n \geq 1$, that satisfies a certain nonlinear boundary condition. We establish our existence results by the method of sub-supersolutions.

1. Introduction

In this paper, we consider a population dynamics model with strong Allee effect and nonlinear boundary conditions, namely

$$u_t = \mathrm{d}\Delta u + a(x)u + b(x)u^2 - m(x)u^3 - \mathrm{ch}(x); \quad \Omega \tag{1.1}$$

$$\mathrm{d}\alpha(x,u)\frac{\partial u}{\partial \eta} + [1 - \alpha(x,u)]u = 0; \quad \partial\Omega \tag{1.2}$$

where Ω is a bounded domain in $\mathbb{R}^n$ with $n \geq 1$, Δ is the Laplace operator, d is the diffusion coefficient, a, b, m are C^μ (Holder continuous) functions such that $b(x), m(x)$ are strictly positive on $\overline{\Omega}$ with $a(x)$ negative at least for some $x \in \Omega$ (strong Allee effect), $c \geq 0$ is the harvesting parameter, $h(x) : \overline{\Omega} \longrightarrow \mathbb{R}$ is a C^1 function, $\frac{\partial u}{\partial \eta}$ is the outward normal derivative, and $\alpha(x,u) : \overline{\Omega} \times \mathbb{R} \longrightarrow [0,1]$ is a C^1 function nondecreasing in u. This type of reaction–diffusion equation has been employed to describe the spatiotemporal distributaries and abundance of organisms living in a patch, Ω. The typical representation of such equations is given by

$$u_t = \mathrm{d}\Delta u + u\tilde{f}(x,u); \quad \Omega \tag{1.3}$$

where $u(t,x)$ denotes the population density and $\tilde{f}(x,u)$ is the per capita growth rate affected by the heterogeneous environment. Skellam first studied such ecological models in his pioneering work, [1]. Previously, Kolomogoroff, Petrovsky, and Piscounoff analyzed similar models in [2]. The classic example is Fisher's equation with $\tilde{f}(x,u) = (1-u)$, first studied by Skellam in [1]. Subsequently, reaction–diffusion models have helped describe spatiotemporal phenomena in other disciplines such as physics, chemistry, and biology (see [3–7]). The logistic growth rate, $\tilde{f}(x,u) = a(x) - b(x)u$, has

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