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Fixed point theorems for single-valued and multi-valued maps

S.L. Singh, S.N. Mishra *

School of Mathematical & Computational Sciences, Department of Mathematics, Walter Sisulu University, Nelson Mandela Drive, Mthatha 5117, South Africa

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ABSTRACT

Coincidence and fixed point theorems for single-valued and multi-valued maps generalizing recent results of Suzuki and Kikkawa are obtained. Various applications, including the existence of common solutions of certain functional equations are presented.

1. Introduction

The classical Banach contraction theorem has recently been generalized by Suzuki [1] in the following way.

Theorem 1.1. *Let* (X, d) *be a complete metric space and let* S *be a self-map of* X*. Define a nonincreasing function* θ *from* $[0, 1)$ *onto* $(\frac{1}{2}, 1]$ *by*

$$\theta(r) = \begin{cases} 1 & \text{if } 0 \le r \le \frac{1}{2}(\sqrt{5}-1) \\ \frac{1-r}{r^2} & \text{if } \frac{1}{2}(\sqrt{5}-1) \le r \le 2^{-\frac{1}{2}} \\ \frac{1}{(1+r)} & \text{if } 2^{-\frac{1}{2}} \le r < 1. \end{cases} \tag{TS}$$

Assume that there exists $r \in [0, 1)$ *such that*

$$\theta(r)d(x, Sx) \le d(x, y) \text{ implies } d(Sx, Sy) \le rd(x, y) \quad \text{for all } x, y \in X. \tag{SC}$$

Then S *has a unique fixed point.*

Now onwards, any self-map S *satisfying (SC) will be called Suzuki contraction and the above theorem as the Suzuki contraction theorem.*

Theorem 1.1 and its further outcomes [2,3] are important contributions to metric fixed point theory. Indeed, [2, Theorem 2] presents a significant generalization of the well-known multi-valued contraction theorem due to Nadler, Jr. [4] (see also [5,6]). On the other hand, Goebel's coincidence theorem [7] has been extended to various settings

* Corresponding author. Tel.: +27 47 5022231; fax: +27 47 5022725.
E-mail addresses: vedicmri@gmail.com (S.L. Singh), smishra@wsu.ac.za (S.N. Mishra).

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