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Ground-state solutions for the electrostatic nonlinear Klein–Gordon–Maxwell system ☆

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ABSTRACT

In this paper, we study the nonlinear Klein–Gordon equation coupled with the Maxwell equation in the electrostatic case:

$$\begin{cases} -\Delta u + [m^2 - (e\phi + \omega)^2]u = f(u), & \text{in } \mathbb{R}^3, \\ \Delta\phi = e(e\phi + \omega)u^2, & \text{in } \mathbb{R}^3, \end{cases} \tag{P}$$

where $m, e, \omega > 0$. Benci and Fortunato (2002) [3] and D'Aprile and Mugnai (2004) [6], showed that, for any $u \in H^1(\mathbb{R}^3)$, the second equation of problem (P) has a unique solution $\phi_u \in D^{1,2}(\mathbb{R}^3)$, the map $\Lambda : u \in H^1(\mathbb{R}^3) \mapsto \phi_u \in D^{1,2}(\mathbb{R}^3)$ is continuously differentiable, and $\phi_u \in [-\omega/e, 0]$. Furthermore, we prove that

$$\max\left\{-\frac{\omega}{e} - \phi_u, \phi_u\right\} \le \psi_u \le 0,$$

where $\psi_u = \Lambda'(u)[u]/2$. Then, we consider the ground-state solution of problem (P) with $f(u) = |u|^{p-2}u$, $2 < p < 6$.

1. Introduction

Recently, the existence of solutions for the following Klein–Gordon–Maxwell systems:

$$\begin{cases} -\Delta u + [m^2 - (e\phi + \omega)^2]u = f(u), & \text{in } \mathbb{R}^3, \\ \Delta\phi = e(e\phi + \omega)u^2, & \text{in } \mathbb{R}^3, \end{cases} \tag{P}$$

where $m, e, \omega \in \mathbb{R}, f : \mathbb{R} \to \mathbb{R}$ is a continuous function, and $f(0) = 0$, have been studied in several works (see [1–6] and the references therein). (P) may describe a standing wave of the nonlinear Klein–Gordon equation interacting with an electromagnetic field in the electrostatic case (for more details, see [3,6]).

Solutions of (P) (u, ϕ) will be sought in $H^1(\mathbb{R}^3) \times D^{1,2}(\mathbb{R}^3)$ as critical points of the following functional:

$$\Psi(u, \phi) = \frac{1}{2}\int_{\mathbb{R}^3} |\nabla u|^2 \,dx - \frac{1}{2}\int_{\mathbb{R}^3} |\nabla\phi|^2 \,dx + \frac{1}{2}\int_{\mathbb{R}^3} [m^2 - (\omega + e\phi)^2]u^2 \,dx - \int_{\mathbb{R}^3} F(u)dx,$$

where $F(t) = \int_0^t f(s)ds$. It is easy to see that $\Psi \in C^1(H^1(\mathbb{R}^3) \times D^{1,2}(\mathbb{R}^3), \mathbb{R})$ and $(u, \phi) = (0, 0)$ is a trivial solution of (P). In order to find the critical points of $\Psi(u, \phi)$, two difficulties have to be overcome. The first difficulty is that Ψ is strongly indefinite, i.e. it is unbounded both from below and from above on infinite-dimensional subspaces. The second difficulty is that the embedding of $H^1(\mathbb{R}^3)$ into $L^q(\mathbb{R}^3)$ for $2 \le q \le 2^*$ is not compact.

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