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Existence and regularity of the solution of a mixed boundary value problem for the Keldysh equation with a nonlinear absorption term

Xu Zhonghai [a,*], Feng Zhenguo [b], Zheng Jiashan [a]

[a] Science College of Northeast Dianli University, Jilin 132013, China
[b] School of Mathematical Sciences, Fudan University, Shanghai 200433, China

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ABSTRACT

The Keldysh equation is a more general form of the classic Tricomi equation from fluid dynamics. Its well-posedness and the regularity of its solution are interesting and important. The Keldysh equation is elliptic in $y > 0$ and is degenerate at the line $y = 0$ in R^2. Adding a special nonlinear absorption term, we study a nonlinear degenerate elliptic equation with mixed boundary conditions in a piecewise smooth domain—similar to the potential fluid shock reflection problem. By means of an elliptic regularization technique, a delicate a priori estimate and compact argument, we show that the solution of a mixed boundary value problem of the Keldysh equation is smooth in the interior and Lipschitz continuous up to the degenerate boundary under some conditions. We believe that this kind of regularity result for the solution will be rather useful.

1. Introduction

The following equation (see [1–3]):

$$y^m u_{xx} + u_{yy} = f(x, y, u) \tag{1.1}$$

is a Keldysh equation when $f(x, y, u) = 0$. The Keldysh equation is elliptic in $y > 0$ and is hyperbolic in $y < 0$ if m is odd; the line $y = 0$ is a degenerate line in R^2. If $m = 1$, the principal part of Eq. (1.1), i.e. $yu_{xx} + u_{yy} = 0$, is called the Tricomi equation. In this paper, we study a mixed boundary value problem for the Keldysh equation with a nonlinear term: $f(x, y, u) = u^p$ $(p > 1)$ in the following Lipschitz domain Ω:

$$\Omega = \{(x, y) : 0 < x < 1, 0 < y < 1\}.$$

Clearly, in Ω, Eq. (1.1) is elliptic and the line $y = 0$ is a degenerate line.

In [3], Keldysh points out that the cases $m < 2$ and $m \geq 2$ are fundamentally different. We only discuss the case $m \geq 2$ in this paper; the case $0 < m < 2$ will be discussed in a forthcoming paper.

Here we should point out that our Eq. (1.1), the boundary conditions and the regularity of the boundary do not fit into the following form discussed in [4–6]:

$$\begin{aligned} &-\sum_{i,j=1}^{n} a_{ij}(x)\partial_{ij}^2 u + \sum_{i=1}^{n} b_i(x)\partial_i u + c(x)u = f(x), \quad x \in Q, \\ &u|_{\Sigma_2 \cup \Sigma_3} = g, \end{aligned} \tag{1.2}$$

* Corresponding author.
E-mail address: xuzhonghai@163.com (Z. Xu).

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