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Nontrivial periodic solutions for strong resonance Hamiltonian systems via a local linking theorem☆

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ABSTRACT

In this paper, we first establish an existence result of critical points for a class of functionals defined on Hilbert spaces by using a local linking idea. Then as an application of the existence result, we obtain the existence of periodic solutions of strong resonance Hamiltonian systems which are asymptotically linear both at infinity and at origin.

1. Introduction

We are concerned in this paper with the existence of nontrivial periodic solutions for the following Hamiltonian systems

$$\dot{x}(t) = JH'(t, x), \tag{1.1}$$

where $H \in C^1(\mathbb{R}^{2N} \times \mathbb{R}, \mathbb{R})$ is 2π-periodic in t, H' denotes the gradient of H with respect to x and

$$J = \begin{pmatrix} 0 & -I_N \\ I_N & 0 \end{pmatrix}$$

is the standard symplectic matrix, where I_N is the identity matrix in $\mathbb{R}^N$.

We assume that there exist two $2N \times 2N$ symmetric, continuous 2π-periodic matrix functions $B_\infty(t)$ and $B_0(t)$ such that

$$|H'(t, x) - B_\infty(t)x| = o(|x|), \quad \text{as } |x| \to \infty, \text{ uniformly in } t, \tag{1.2}$$

$$|H'(t, x) - B_0(t)x| = o(|x|), \quad \text{as } |x| \to 0, \text{ uniformly in } t. \tag{1.3}$$

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