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Optimality conditions and the basic constraint qualification for quasiconvex programming

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ABSTRACT

In this paper, we consider optimality conditions and a constraint qualification for quasiconvex programming. For this purpose, we introduce a generator and a new subdifferential for quasiconvex functions by using Penot and Volle's theorem.

1. Introduction

We consider the following minimization programming problem:

$$\begin{cases} \text{minimize } f(x), \\ \text{subject to } g_i(x) \le 0, \quad \forall i \in I, \end{cases}$$

where I is an arbitrary set, and f and g are extended real-valued functions from the locally convex Hausdorff topological vector space X. When f and g_i are convex and $x_0 \in A = \{x \in X \mid \forall i \in I, g_i(x) \le 0\}$, the following equivalence relation holds under some constraint qualifications:

$$f(x_0) = \inf_{x \in A} f(x) \iff \exists \lambda \in \mathbb{R}_+^{(I)} \text{ s.t. } 0 \in \partial f(x_0) + \sum_{i \in I} \lambda_i \partial g_i(x_0),$$

where $\mathbb{R}_+^{(I)} = \{\lambda \in \mathbb{R}^I \mid \forall i \in I, \lambda_i \ge 0, \{i \in I \mid \lambda_i \ne 0\} : \text{finite}\}$. The constraint qualifications for this optimality condition have been studied by many researchers. Recently, the basic constraint qualification (the BCQ) was proposed by Li et al. [1]. The BCQ is said to be the weakest constraint qualification for this optimality condition because the BCQ and this optimality condition are equivalent.

The purpose of this paper is to generalize the result of [1] for quasiconvex programming. In quasiconvex optimization, Penot and Volle [2] reported an interesting result whereby a lower semi-continuous quasiconvex function consists of a supremum of some family of lower semi-continuous quasiaffine functions. This result is fundamental and useful for our purpose.

In the present paper, we consider optimality conditions and the basic constraint qualification for quasiconvex programming. By using Penot and Volle's theorem, we introduce the notion of a "generator" and a new subdifferential for quasiconvex functions, and investigate generalized results reported in previous studies.

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