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A regularity criterion for the 3D magneto-micropolar fluid equations in Triebel–Lizorkin spaces

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ABSTRACT

We consider the regularity criterion for the 3D magneto-micropolar fluid equations in Triebel–Lizorkin spaces. It is proved that if $\nabla \boldsymbol{u} \in L^p\left(0, T; \dot{F}^0_{q,2q/3}\right)$ with

$$\frac{2}{p} + \frac{3}{q} = 2, \quad 3/2 < q \leq \infty,$$

then the solution remains smooth in $(0, T)$. As a corollary, we obtain the classical Beal–Kato–Majda criterion, that is, the condition

$$\nabla \times \boldsymbol{u} \in L^1(0, T; \dot{B}^0_{\infty,\infty}),$$

ensures the smoothness of the solution.

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1. Introduction

We consider the magneto-micropolar fluid equations in $\boldsymbol{R}^3$:

$$\begin{cases} \partial_t \boldsymbol{u} + (\boldsymbol{u} \cdot \nabla)\, \boldsymbol{u} - (\mu + \chi)\Delta \boldsymbol{u} - (\boldsymbol{b} \cdot \nabla)\, \boldsymbol{b} + \nabla\left(p + |\boldsymbol{b}|^2\right) - \chi \nabla \times \boldsymbol{\omega} = 0, \\ \partial_t \boldsymbol{\omega} - \gamma \Delta \boldsymbol{\omega} - \kappa \nabla \mathrm{div}\, \boldsymbol{\omega} + 2\chi \boldsymbol{\omega} + \boldsymbol{u} \cdot \nabla \boldsymbol{\omega} - \chi \nabla \times \boldsymbol{u} = 0, \\ \partial_t \boldsymbol{b} - \nu \Delta \boldsymbol{b} + (\boldsymbol{u} \cdot \nabla)\, \boldsymbol{b} - (\boldsymbol{b} \cdot \nabla)\, \boldsymbol{u} = 0, \\ \nabla \cdot \boldsymbol{u} = \nabla \cdot \boldsymbol{b} = 0, \\ \boldsymbol{u}(x, 0) = \boldsymbol{u}_0(x), \qquad \boldsymbol{\omega}(0, x) = \boldsymbol{\omega}_0(x), \qquad \boldsymbol{b}(0, x) = \boldsymbol{b}_0(x). \end{cases} \tag{1.1}$$

Here $\boldsymbol{u} = \boldsymbol{u}(x, t)$ represents the velocity field, $\boldsymbol{b} = \boldsymbol{b}(x, t)$ represents the magnetic field, $\boldsymbol{\omega} = \boldsymbol{\omega}(x, t)$ represents the micro-rotational velocity; p denotes the hydrodynamic pressure; $\mu > 0$ is the kinematic viscosity, $\chi > 0$ is the vortex viscosity, $\kappa > 0$ and $\gamma > 0$ are spin viscosities, $1/\nu$ (with $\nu > 0$) is the magnetic Reynolds; while $\boldsymbol{u}_0, \boldsymbol{b}_0, \boldsymbol{\omega}_0$ are the corresponding initial data with $\mathrm{div}\, \boldsymbol{u}_0 = \mathrm{div}\, \boldsymbol{b}_0 = 0$.

This system is of interest for various reasons. For example, it includes some well-known equations, say the Navier–Stokes equations ($\boldsymbol{\omega} = \boldsymbol{b} = 0$) and the MHD equations ($\boldsymbol{\omega} = 0$). Moreover, it has similar scaling properties and energy estimates as the Navier–Stokes and MHD equations. We believe that the regularity theory of system (1.1) can improve the understanding of the Navier–Stokes and MHD equations.

System (1.1) was first proposed by Galdi and Rionero [1]. The existence of global-in-time weak solutions were then established by Rojas-Medar and Boldrini [2], while the local strong solutions and global strong solutions for the small initial

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