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On the Conley index in the invariant manifolds

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ABSTRACT

Let ϕ be a flow on a manifold M and assume that $N \subset M$ is an invariant manifold. The aim of this note is to compare the Conley indices of an isolated invariant set $S \subset N$ with respect to the flow ϕ and the flow ϕ restricted to N.

1. Introduction

The motivation for this paper comes from [1–5] where the Conley index theory and the fixed point index theory were used in the study of the problems related to permanence in ecological systems. The phase spaces in ecological systems are usually manifolds M with boundary and the natural question that arises is that of whether there is a point $x \in M \setminus \partial M$ such that the omega limit set of x is nonempty and contained in ∂M (see [6,7,2,8]). The results of [1] showed that the Conley index theory seems to be a very useful tool in the study of permanence problems. In this paper we extend the approach presented in [1,2,4,5] to the case where the invariant submanifold is not necessarily of codimension 1. Namely, the context of this note is the following. Let $\phi : \mathbb{R} \times M \to M$ be a flow on a manifold M and $N \subset M$ an invariant submanifold. Assume that $S \subset N$ is an isolated invariant set for the flow ϕ. Then S is also an isolated invariant set for the flow ϕ restricted to N. The natural question that arises that of how the Conley indices of S with respect to ϕ and $\phi|_N$ are related. We present a result in this direction under the assumption that S is of attracting or repelling type with respect to the manifold N.

2. Isolating blocks and the Conley index

Let M be a topological manifold of dimension m (without boundary). A continuous map $\phi : \mathbb{R} \times M \to M$ is called a flow if

$$\phi(0, x) = x, \qquad \phi(s, \phi(t, x)) = \phi(s + t, x), \quad s, t \in \mathbb{R}, \ x \in M.$$

In the sequel we frequently use the following notation: we write $\phi_t(x)$ instead of $\phi(x, t)$ and if $W \subset M$ and $J \subset \mathbb{R}$ then we write $\phi(J, W)$ instead of $\phi(J \times W)$. The sets

$$\phi(x) := \phi(\mathbb{R}, x),$$
$$\phi^+(x) := \phi([0, +\infty), x),$$
$$\phi^-(x) := \phi((-\infty, 0], x),$$

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